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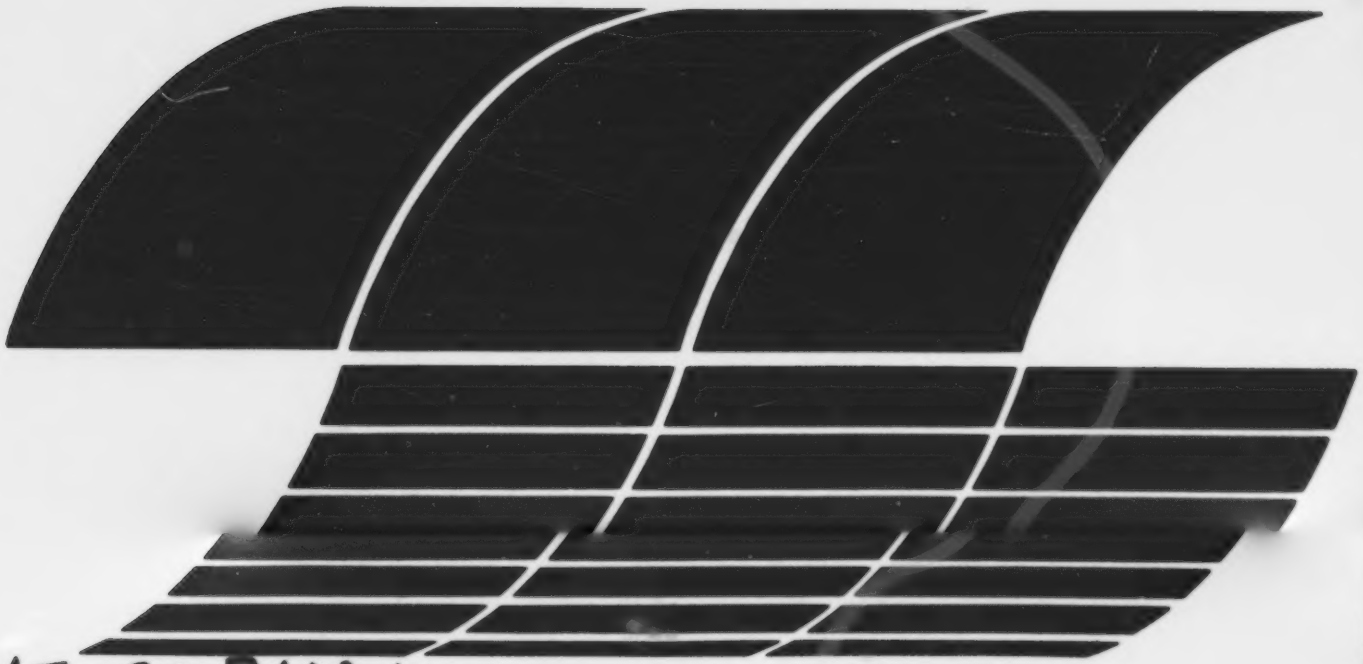
Theoretical Investigation of Inlet Characteristics for Personal Aerosol Samplers

Interagency Energy/Environment R&D Program Report



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THEORETICAL INVESTIGATION OF INLET CHARACTERISTICS FOR PERSONAL AEROSOL SAMPLERS

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U.S. DEPARTMENT OF HEALTH AND HUMAN SERVICES
Public Health Service
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ABSTRACT

To keep the working environment as healthful as possible the permissible exposure to dust particles has to be decreased. Lower permissible exposure makes the representativeness of the sample more important and methods to estimate the sampling errors are necessary. A theoretical model and a computer program system has been developed to estimate such errors.

The computer program can handle various inlet geometries such as circular tube and parallel plates. Samplers whose face is not perpendicular to the ambient flow are simulated by a line sink. The model accounts for inertial and sedimentation effects on particle motion.

The results obtained by use of the computer program system agree very well with the experimental data reported in the literature.

CONTENTS

Abstract. iii
List of Figures v
List of Tables. vii
Introduction. 1
Literature Survey 3
Theoretical Model 12
Method of Solution. 33
Results and Discussion. 50
References. 75
Appendixes
A. Derivation of Boundary Condition at Section II. 78
B. Stream Function for Uniform Sink Strength Distribution. 80
C. Stream Function for Triangular Sink Strength Distribution. 82
D. Computer Program Listing. 84
E. Computer Program System User Manual 110
Glossary. 125

FIGURES

1. Comparison of theoretical and experimental effects of anisokinetic sampling. 6
2. Comparison of experimental and theoretical data taken from various authors. 8
3. Flowfield in the case of sampling a stationary fluid into a tube 9
4. Particle trajectories at $U/U_0 = 25$, $K = 0.3$ 10
5. Modeling strategy 13
6. Coordinate system for solving Equation (14) 15
7. Nomenclature of flow region for two parallel plates 17
8. Nomenclature of flow boundary for circular tube 20
9. Line sink distribution along face plane of the sampler. . . 22
10. Triangular source/sink strength distribution. 24
11. Coordinate system for electrostatic force calculation . . . 30
12. Rectangular mesh geometry 34
13. Unequal grid size 36
14. Various inlet geometries. 37
15. Flow field boundary and grid point layout 45
16. Boundary conditions for circular inlet. 46
17. C/C_0 vs. Stokes's number $U/U_0 = 1.2$ thin-walled circular tube 51
18. C/C_0 vs. Stokes's number $U/U_0 = 2.0$ thin-walled circular tube, $\alpha = 90^0$. 52
19. Sampling bias for thin-walled circular tube facing the stream, velocity ratio = 0.75 53
20. Sampling bias for thin-walled circular tube facing the stream, velocity ratio = 0.375. 54
21. Sampling bias for thin-walled circular tube facing the stream, velocity ratio = 0.1875 55
22. Sampling bias for thin-walled circular tube facing the stream, velocity ratio = 0.0938 56
23. Trajectory of particles, $U = 0.375$, $\alpha = 90^0$, $K = 0.01$ . . . 57
24. Trajectory of particles, $U = 0.375$, $\alpha = 90^0$, $K = 0.3$ . . . 58
25. Trajectory of particles, $U = 0.375$, $\alpha = 90^0$, $K = 3.0$ . . . 59
26. Trajectory of particles, $U = 0.375$, $\alpha = 60^0$, $K = 0.01$ . . . 60
27. Trajectory of particles, $U = 0.375$, $\alpha = 0^0$, $K = 0.01$ . . . 61
28. C/C_0 vs. orientation angle for $U = 0.375$ and lip depth of 0.2. 63
29. C/C_0 vs. Stoke's number for $\alpha = 90^0$, square inlet, ZLIP = 0. 64
30. C/C_0 vs. Stoke's number for $\alpha = 90^0$, square inlet, ZLIP = 0. 65

READEX FILMS

31. Stokes number vs. C/C_0 for U = 0.375, square inlet, ZLIP = 0. 66
32. C/C_0 vs. α for U = 0.375, square inlet, ZLIP = 0. 67
33. Effect of bias on the distribution, $\alpha = 90^0$, square inlet, ZLIP = 0 68
34. C/C_0 vs. U/U_0 for square inlet facing the stream. 70
35. C/C_0 vs. K for various thicknesses of wall with circular inlet facing the stream, $U/U_0 = 0.375$. 71
36. Comparison of theoretical results with experimental data of S. Badzioch (1959), circular tube facing the stream. 72
37. Parameter description for inlet geometry and orientation 110

TABLES

1. Permissible radii of tubes (cm) for sampling aerosols in calm conditions 11
2. Typical values of Stokes number K, Peclet number Pe, Froude number Fr for unit density spheres 31
3. Parameter description and physical significance 47

INTRODUCTION

Almost all methods of aerosol characterization begin with aerosol sampling--the capture and transport of aerosols to a characterizing instrument of some kind. The most important aspect of sampling is the representativeness of the sample. Representativeness exists when both the sample and the aerosol from which it was drawn are identical with respect to concentration, particle size distributior, and chemical composition.

As early as 1911, Brady and Touzalin[1] predicted and experimentally verified the possibility of obtaining nonrepresentative samples of particulate matter by a sampler. Since then, the experimental data of numerous investigators have confirmed the impostance of sampling procedures. Few, however, have made attempts to establish a theoretical basis for estimating sampling error.

To keep the working environment as safe as possible the permissible exposure to particles is made less and less. Lower permissible exposure makes the representativeness of the sample more important, and methods to estimate the sampling errors are necessary.

The quality of air in an industrial environment is evaluated by use of sampling methods. The recent emphasis on personal sampling has produced a trend to wear small battery operated samplers. The usual approach is to attach a filter in a two or three piece cassette to the workers breathing zone and connecting the cassette to a belt mounted, battery operated pump with an appropriate length of tubing. Personal respirable dust samplers consist of a cyclone fitted to the sampling head that may be attached to the worker's clothing near his breathing zone. The cyclone is designed to closely approximate the AEC-ACGIH respirable dust curves. Some of the other commonly used samplers are horizontal elutriator, open faced filter, closed face filter, etc. The inlet geometries of the above mentioned samplers vary widely, from a square inlet in a cyclone to a circular inlet in a closed face filter.

The goal of this research program is to develop a computer program system for predicting the sampling errors for various inlet geometries encountered in personal sampling.

The program involves the following tasks:

Task 1: Literature Survey
Task 2: Theoretical Simulation of Fluid Flow
Task 3: Theoretical Simulation of Particle Motion

Fluid flow simulation required the modelling of the following:

(a) Sampling procedures and conditions for obtaining the best results differ markedly for sampling from flowing and stationary environments. Hence, a model for the environment from which the sample is drawn is very important for meaningful results.

(b) The disturbance created by introducing a sampling inlet into the above known environment establishes a new pattern of fluid flow in the vicinity of the sampler inlet. The velocity of suction of the sample further changes the fluid dynamics of the sampling. Hence, a model for the fluid flow in and around the sampling inlet has to be developed.

To estimate the performance of the sampler inlet, the effect of flow field on particle transport has to be studied. Transport of particles depends on the fluid flow, as well as such factors as particle size, density, and shape. So a model that would enable us to calculate the transport of aerosols into the probe has to be developed. This would complete the model for effectively predicting the errors that occur during the sampling.

LITERATURE SURVEY

"Literature Search" was initiated upon commencement of this project with the help of IITRI's Computer Search Center. The following data bases were searched for pertinent works in "Inlet Characteristics of Aerosol Samplers."

(1) NTIS 1964-1978
(2) APTIC (Air Pollution Abstracts) 1966-1978
(3) Compendex (Engineering Index) 1970-1978
(4) CAC (Chemical Abstracts) 1969-1978

A total of 354 citations were obtained from the computer search. Both theoretical and experimental works that are directly connected to the program were reviewed. Two substantially different types of aerosols are sampled in practice--flowing and stationary aerosols.

(a) Sampling from flowing aerosols is encountered in the study of ducted aerosols and atmospheric aerosols in the presence of wind. Lappel and Shepperd[2] were the first to present a theoretical approach and proposed an equation for assessing the order of magnitude of anisokinetic errors. DallaValle[3] suggested the use of experimentally determined velocity contours as a possible means for determining the errors.

There was a growing interest in this problem in the fifties and only the two methods proposed by Watson[4] and Badzioch[5,6] appear to be practical. The above methods allow the determination of deviations in measured concentration of particulates. Watson[4] gave the following equation for estimating the deviation.

$$\frac{C}{C_o} = \frac{U_o}{\bar{U}} \left[1 + f(K) \left(\frac{\bar{U}}{U_o}^{\frac{1}{2}} - 1 \right) \right] \qquad (1)$$

where

$f(K)$ = unknown function of K and is evaluated by experimental data
C = measured concentration of particles (#/cc)
C_o = actual concentration of particles (#/cc)
U_o = stream velocity (cm/sec)
$\bar{U}$ = the mean air velocity at the sampling orifice
K = dimensionless inertial parameter called Stokes number

Badzioch investigated the efficiency of collection of gas-borne particles by an aspirated sampling nozzle. The efficiency was shown theoretically to depend on (a) the ratio of the velocity of aspiration into the sampling nozzle to the velocity of the undisturbed gas stream, and (b) the ratio of a length representing the distance of disturbance upstream of the nozzle to the "range" of a particle. The "range" is defined as the distance a particle would travel, before coming to rest, if projected into still gas with a velocity equal to that of the gas stream. In the range of conditions investigated experimentally, which included nozzles of 0.65 to 1.90 cm diameter aspirating from turbulent gas streams, it is found that the length representing the upstream disturbance is a function of the diameter of the nozzle.

He gave the following expression for estimation of sampling error:

$$\frac{C}{C_o} = \alpha \frac{U_o}{\overline{U}} + (1-\alpha) \qquad (2)$$

where

$\alpha = [1 - \exp(-D/\lambda)]/(D/\lambda)$

$\lambda = KL$ = range of a particle

D = assumed distance from the plane of nozzle exit at which the streamlines start either diverging or converging and is a function of L. The value of D was deduced from experiments.

Both of the above models are semi-empirical, using experimentally derived parameters to bridge gaps in theoretical deductions.

Levin was the first to consider the inertial lag of particles with the particles flowing into a point sink in the field of uniform wind. His theoretical results are valid if

$$\frac{\overline{U}}{U_o} < 64K \qquad (3)$$

is fulfilled, where $\overline{U}$ is the suction velocity of the sample (cm/sec), U_o is free stream velocity, and K is the Stokes number ($U_o\tau/L$). According to Levin, if particle sedimentation is ignored, the aspiration coefficient due to inertia $A_i = C/C_o$, where C is the measured concentration (#/cm^3) and C_o is the actual concentration (#/cm^3), is determined as follows:

$$A_i = 1 - 3.2 \frac{U_o}{\overline{U}} K + 0.44 \frac{U_o}{\overline{U}} K^2 \qquad (4)$$

Where $\overline{U}/U_o$ values from 4.0 to 8.0 the experimental results were in good agreement with Equation 4.

In the late fifties and early sixties there were a number of experimental studies on the effect of probe shape on the sampling accuracy and directional dependence of samplers[8,9].

Vitols[10,11] calculated, by means of a computer, the flow field for an aspirated nozzle for various values of $\overline{U}/U_o$ ignoring the turbulence and viscosity of the fluid. Particle trajectories were computed for different values of the Stokes number, K. Gravitational force on the particles was not considered. The graphs A_i vs. $\overline{U}/U_o$ were obtained using the Stokes number as the parameter. A typical curve is shown in Figure 1. The lowest value of K used was large. For large values of K, the particle trajectories are nearly straight. Their motion is nearly independent of the flow lines and $A_i = U_o/\overline{U}$ because the efficiency of capture of the particles is 100% for large K values. These theoretical error estimates compared well with the experimental data of Badzioch which have a very large scatter in values of A_i. According to Davies,[12,13] when the wind velocity $U_o \leq \overline{U}/5$, the aspiration coefficient can be estimated from the sampling criteria for stationary systems. Whenever this condition is not met, the aspiration coefficient can be calculated by using Levin's[7] solution for A_i. Because Levin's solution is based on a point sink that draws particles from a uniform stream, it is applicable only to sampling tubes of small radii. According to Davies, the maximum diamter, D, of the tube is:

$$\frac{D}{2} = \frac{\left(\frac{F}{4\pi U_o}\right)^{\frac{1}{2}}}{5} \tag{5}$$

where F is the rate of suction of the sampler.

For larger sampling tubes Davies gave an empirical relation based on the experimental data and A_i is determined as follows:

$$A_i = \frac{C}{C_o} = 1 + \left(\frac{U_o}{\overline{U} - 1}\right) f(K) \tag{6}$$

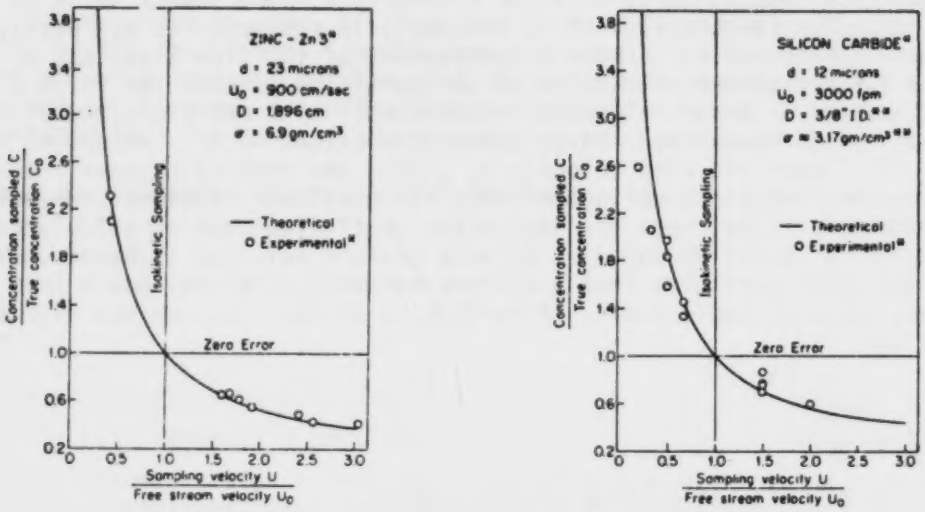
where K is the particle Stokes number and

$$f(K) = 1 - \frac{1}{1 + 2K} \tag{7}$$

Belyave and Levin[14] showed that the observed aspiration coefficient A_e could not be identified with purely inertial aspiration A_i that had been the main subject of the earlier studies (1-13). Taking into account the rebound of the particle and its deposition in the sampling tube, A_e will be a function of A_i, A_d (the coefficient characterizing the particle concentration decrease in a sample caused by deposition in the inlet channel) and A_r (the coefficient which depends on the particle rebound from the front edge of the sampling nozzle and their subsequent aspiration into it).

All three coefficients, A_i, A_d, and A_r, strongly depend on nozzle shape. Since the character of this dependence for A_r and A_d has yet to be established, A_r and A_d were assumed to be united and gave an empirical relation for A_i as follows:

$$A_i = 1 + \left(\frac{U_o}{U} - 1\right)\beta \tag{8}$$



(a)

Zinc Sphere, K = 10.5

(b)

Silicon Carbide, K = 4.5

Figure 1. Comparison of theoretical and experimental effects of anisokinetic sampling (from reference [10]).

where β is a non-dimensional function, $\beta \to 0$ as $K \to 0$, and $\beta \to 1$ as $K \to \infty$. All the previous works concluded that β is a function of K only. But in Figure 2, which gives theoretical curves as well as experimental points, β shows a dependence on $\overline{U}/U_o$ and is given as follows:

$$\beta = 1 - \frac{1}{1 + BK} \tag{9}$$

and B, a non-dimensional function, is given by:

$$B = 2 + 0.617 \frac{\overline{U}}{U_o} \tag{10}$$

(b) Sampling from calm air or stationary aerosols has been investigated much less than sampling from flowing aerosols. Figures 3 and 4 demonstrate that the character of the flow fields at the entrance to a tube from a stationary medium and from a stream $0 < U_o/\overline{U} < 1$ is quite different[15].

Levin[16] considered the sampling from stationary aerosols as a flow into a point sink. He developed a relation for A_i, the inertial aspiration coefficient, which is:

$$A_i + 1 - 0.8K + 0.08K^2 + \ldots \tag{11}$$

where $K = \tau(4\ V_s^3/F)^{1/2}$ is a parameter acting as a Stokes number, F is the flow rate into the sink, and V_s is the sedimentation velocity of the particle.

In his well known theory of sampling, Davies[12,13,15,23,33] used the stopping distance of a particle to characterize inertial effects and terminal settling velocity to characterize sedimentation of particles. For inertia to be negligible, the stop distance should be small compared to the radius of the tube D/2. For sedimentation to be negligible, the flow velocity in the probe should be much larger than the terminal settling velocity. Then the complete condition to obtain a true sample is:

$$\left(\frac{F\tau}{4\pi}\right)^{1/3} << \frac{D}{2} << \left(\frac{F}{\pi g \tau}\right)^{1/2} \tag{12}$$

where F is the rate of suction of the sample (cm^3/sec), τ is the relaxation time of particles (sec), and g is acceleration due to gravity (cm/sec^2). Using an arbitrary criteria of 1/5 he arrives at the following condition:

$$5\left(\frac{F\tau}{4\pi}\right)^{1/3} \leq \frac{D}{2} \leq \frac{\left(\frac{F}{\pi g \tau}\right)^{1/2}}{5} \tag{13}$$

Table 1 shows the permissible tube sizes for sampling aerosols as a function of suction rate and particle size. When the tube radius is greater than the lower figure, sampling errors due to inertia are not significant. When the radius is less than the upper limit, sedimentation is not significant. The two criteria can be satisfied simultaneously for the unbracketed entries in the table but not for bracketed entries. Satisfactory samples can be obtained by using the tube size satisfying the lower limit condition and orienting the sampler vertically to eliminate sedimentation.

Agarwal[36] studied the problem of aerosol sampling under calm air conditions by solving the Navier-Stokes equations and equation of variable motion. The study was continued to circular inlets. The sampling efficiency of an inlet was found to depend upon two dimensional parameters, the Stoken Number, K, and the relative velocity, Vs^1. Using an arbitary criteria of 90x efficiency, he arrives at the following condition:

$$2.K.Vs^1 \leq 0.1$$

This criterion is less restrictive than the condition given by equation (13) and provides adequate accuracy.

Ter Kuile[37] made the distinction between "representative sampling" and "comparable sampling". This results in two criteria.

1. The criterion for representative sampling which combines a modification of DAVIES' theory for representative sampling and LEVIN's sampling theory into one criterion which is limited by three physical effects:

- impaction on the wall of the inlet;
- sedimentation on the wall of the inlet;
- dynamic escape from the sampling region.

2. The criterion for comparable sampling requires that the inlet is to point vertically downwards with a filter near the inlet, resulting in a higher efficiency, sharper cut-off limits, better reproducibility and better comparability.

Both these criteria were given in graphs in which the sampling efficiency is a function of a dimensionless particle size number (k_{dQ}), and a tube size number (m_{DQ}). Theoretical limits of the region where the efficiency is more than 90% for representative sampling and more than 94% for comparable sampling are plotted in these graphs. The tube size number only depends on parameters of the sampling device, so that the design of this device determines the nature of the physical limitation. As a result of this, sampling devices can be divided into three classes for which different physical mechanisms limit the aspiration efficiency of large particles.

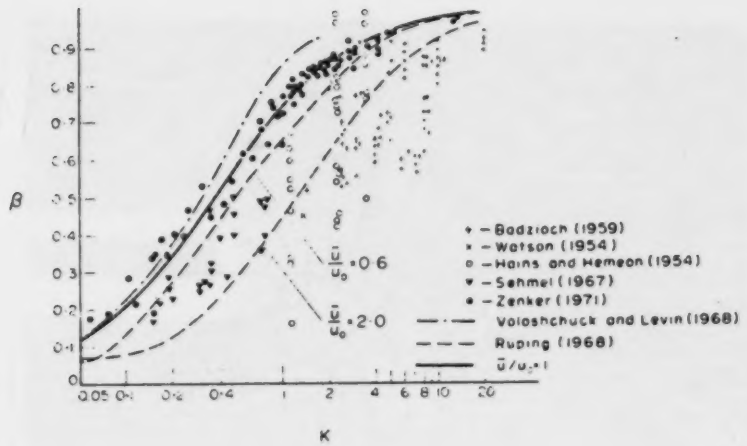


Figure 2. Comparison of experimental and theoretical data taken from papers of various authors (from reference [14]).

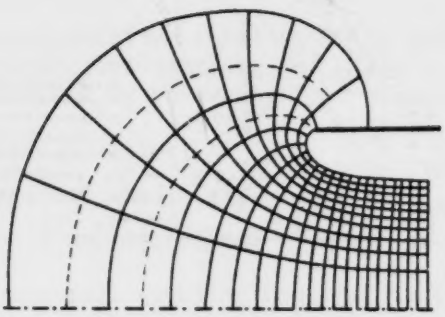


Figure 3. Flow field in the case of sampling a stationary fluid into a tube (from reference [17]).

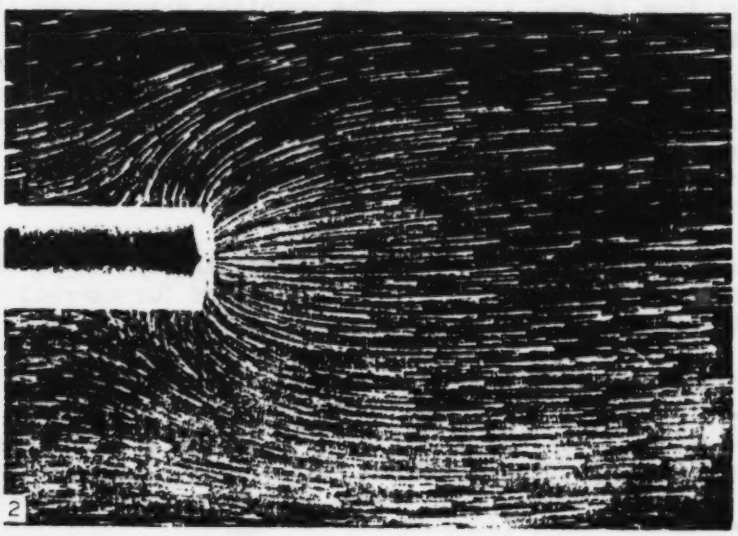


Figure 4. Particle trajectories at $U/U_0 = 25$, $K = 0.3$ (from reference [17]).

Fuchs presented a review of methods of sampling and methods to estimate the bias. During this literature search the author came across a number of experimental studies on the effect of probe shape, orientation and velocity of sampling on entry efficiencies.

Glauberman[9] studied the directional dependence of air samplers using uranium oxide dusts. Two filtration type air samplers were tested. In turbulent air no bias was found due to orientation. In a directional air stream, a sampler head facing into the air stream collected more dust by a factor of two compared to a sampler facing up or down. Schmel[19] investigated particle sampling errors from several sources. The sampling errors were significant for particles as small as 1 micron in some cases. Pickett and Sansone[22] studied the effect of varying inlet geometry on collection characteristics of a 10-millimeter Nylon Cyclone. Samples of coal dust aerosol were collected simultaneously with two filter holders: one designed to conform to Davies' criteria, and one with smaller inlet dimensions corresponding to those of the 10 mm Nylon Cyclone. No differences in mass concentration or size distribution were obtained.

Breslin and Stein[26] considered sampling inlets in calm air. Their results showed that Davies' criteria for inlet conditions for correct sampling are overly restrictive.

Raynor[30] studied the effect on the entrance efficiency of a filter holder caused by variations in the following parameters--air speed, flow rate, angle between the air flow and the filter holder, and particle size. Efficiencies for various particles were determined over a range of angles from 60-120 degrees from horizontal for wind speeds of 100, 200, 400 and 700 cm/sec and filter flow rates of 6.4, 12.7 and 25.4 liters/min. The entrance efficiency varied with all parameters from less than 1% at highest wind speed and lowest flow rate to over 100% at forward angles. Efficiency was lowest with filter holder entrance at right angles to the air stream.

Table 1. Permissible radii of tubes (cm) for sampling aerosols in calm conditions*

Particle diameter, μ	Rate of suction, F (cm^3/sec)					
	1	10	100	1,000	10,000	100,000
1	0.033 – 1.9	0.071 – 6.0	0.15 –19	0.33 –60	0.71 –190	1.5–600
2	0.051 – 1.0	0.11 – 3.2	0.23–10	0.51 –32	1.1 –100	2.3–320
5	0.093 – 0.41	0.20 – 1.3	0.43– 4.1	0.93 –13	2.0 – 41	4.3–130
10	0.15 – 0.21	0.31 – 0.65	0.68– 2.1	1.5 – 6.5	3.1 – 21	6.8– 65
20	(0.23 ~ 0.10)	(0.50 ~ 0.33)	(1.1 ~ 1.0)	2.3 – 3.1	5.0 – 10.3	11.0– 31
50	(0.42 ~ 0.042)	(0.90 ~ 0.13)	(1.9 ~ 0.42)	(4.2 ~ 1.33)	(9.0 ~ 4.2)	(19 ~ 13.3)
100	(0.63 ~ 0.023)	(1.4 ~ 0.071)	(2.9 ~ 0.23)	(6.3 ~ 0.71)	(14.0 ~ 2.3)	(29 ~ 7.1)
200	(0.89 ~ 0.014)	(1.9 ~ 0.037)	(4.1 ~ 0.14)	(8.9 ~ 0.37)	(19 ~ 1.4)	(41 ~ 3.7)
500	(1.26 ~ 0.008)	(2.7 ~ 0.025)	(5.8 ~ 0.08)	(12.6 ~ 0.25)	(27 ~ 0.80)	(58 ~ 2.5)

* From Reference [12]

THEORETICAL MODEL

An overall theoretical model to simulate a personal sampler requires modeling of the following:

- Model for environmental flow patterns
- Model for flow pattern in and around the sampler
- Model for motion of particles

The strategy used to obtain the overall model for personal sampling is given in the form of a block diagram in Figure 5.

ENVIRONMENTAL FLOW PATTERNS: MODEL FOR INDUSTRIAL ENVIRONMENT

Industrial environmental flow patterns are so complex and individual that any one particular model cannot describe the actual field for all types of working environments. Hence, it is necessary to make some simplifying assumptions so that a model can be developed. Different types of flow patterns exist in a variety of working environments.

(1) An environment essentially calm except for microscopic fluctuations. This type of pattern can be found in research laboratories, nuclear reactor plants, etc.

(2) A steady flow exists in a particular direction even though the direction cannot be fixed. The effect of varying directions can be studied by using it as a parameter. This kind of flow pattern will exist in industries which need high ventilation.

(3) In some working environments, the flow volume is so large that the medium is essentially turbulent in nature. In this type of environment the flow does not realize the existence of an object (sampling head, for instance) and barges into it. This kind of flow would be predominant in environments which need very high ventilation, such as mines.

Hence, the model uses the calm air type of sampling for environments of type 1 and a steady flow type for industrial environments of type 2. Type 3 environments could be realized by increasing the flow velocity of the uniform flow of type 2 environments.

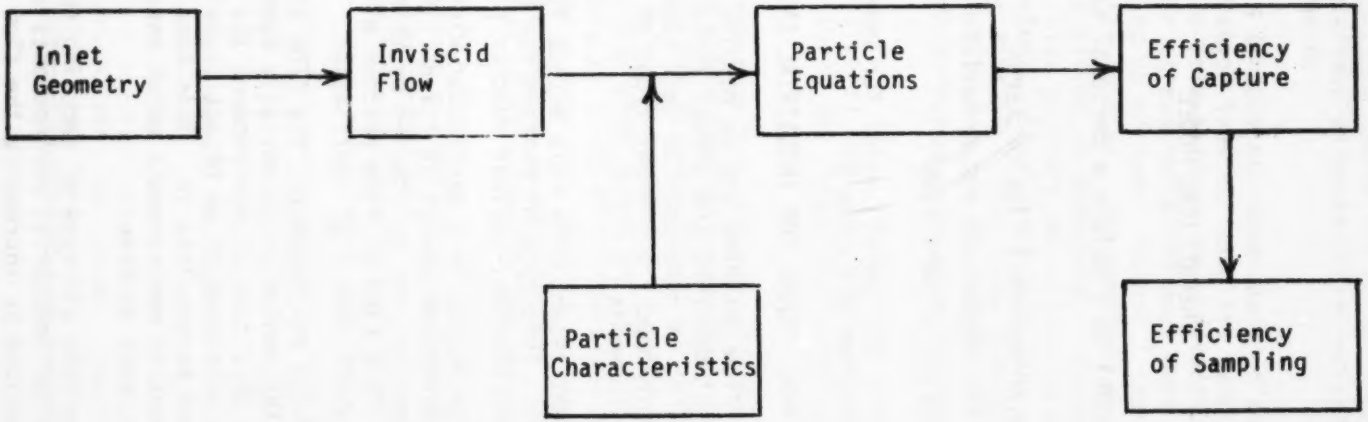


Figure 5. Modeling Strategy

FLOW RATE AND INLET GEOMETRY: MODEL FOR FLOW PATTERN IN AND AROUND THE PROBE

The model for flow pattern in and around the probe is assumed to be given by the inviscid flow pattern. In solving the inviscid problem above, it was assumed the fluid was ideal, without any viscosity as it glides over the walls of the boundary. In reality all fluids have viscosity, hence friction. So the fluid in contact with the wall will be subjected to a no slip condition, that is, the velocity at the wall is zero. This viscous effect extends only to a very small distance from the wall and was not taken into account in this study.

Sampler Facing the Stream

Equations in this section are formulated in the general form so that they can be applied to any given geometry. Ideal fluid flow patterns and velocity distributions can be determined by solving the Laplace equation for the stream function, ψ.[16]

$$\nabla^2 \psi = 0 \tag{14}$$

where ∇^2 is the Laplace operator.

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \text{ for two dimensions}$$

where ∇^2 is the second order central difference operator. The shape of the body determines the boundary conditions on the stream function. General conditions will be:

(1) ψ = constant at the body surface
(2) ψ = 0 at the axis of symmetry
(3) ψ = ψ of the environmental flow pattern upstream and downstream from the body

(15)

The velocity of flow can be calculated from ψ as:

(i) for Cartesian coordinates (x,y,z) (see Figure 6)

$$U_x = \frac{\partial \psi}{\partial y}$$

$$U_y = \frac{\partial \psi}{\partial x} \tag{16}$$

(ii) for cylindrical coordinates (r,z) (see Figure 6)

$$U_r = -\frac{1}{r}\frac{\partial \psi}{\partial y}$$

$$U_z = \frac{1}{r}\frac{\partial \psi}{\partial r} \tag{17}$$

Hence, once ψ is calculated from equation (14) with the use of conditions (15) then the velocity distribution can be found from either (16) or (17).

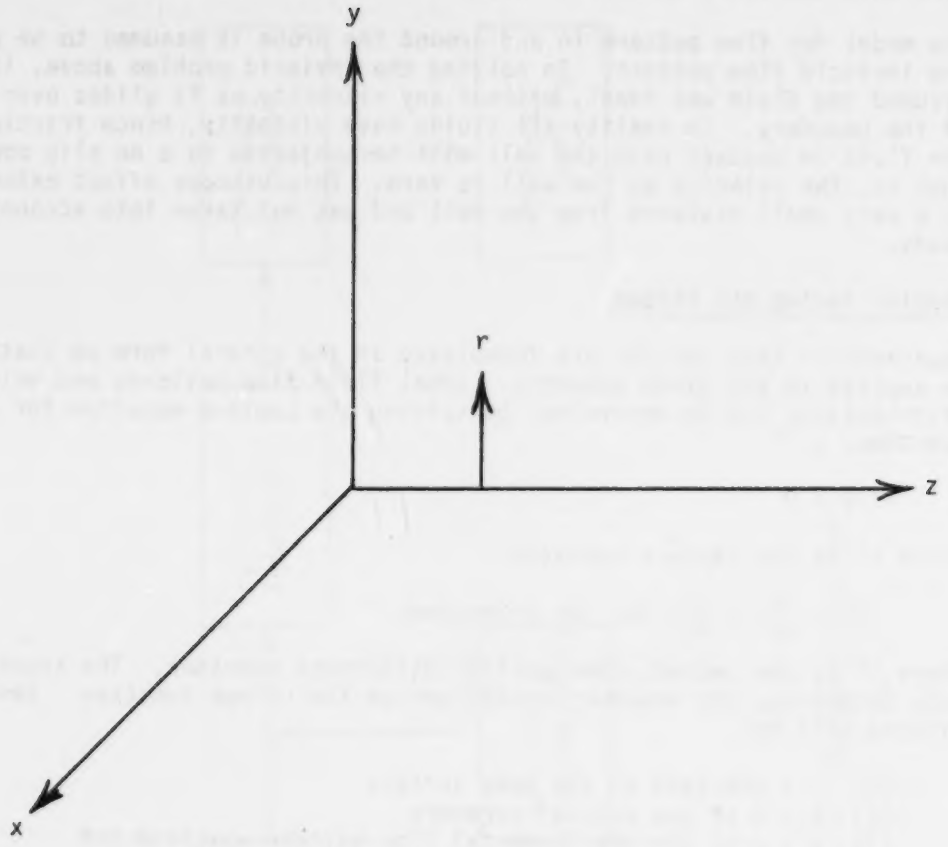


Figure 6. Coordinate system for solving Equation (14).

The stream function ψ does not exist for three dimensional flows. That is, there is no function ψ such that a isoline is a stream line. But for a solenoidal vector field (where $\bar{U}$ satisfies the 3-D continuity equation, $\nabla \cdot \bar{U} = 0$), a so-called vector potential $\bar{\psi} = \psi_x \bar{i} + \psi_y \bar{j} + \psi_z \bar{k}$ does exist, such that the velocity components are given by:

$$U_x = \frac{\partial \psi_z}{\partial y} - \frac{\partial \psi_y}{\partial z}$$

$$U_y = \frac{\partial \psi_z}{\partial x} - \frac{\partial \psi_x}{\partial z} \qquad (18)$$

$$U_z = \frac{\partial \psi_y}{\partial x} - \frac{\partial \psi_x}{\partial y}$$

and $\bar{\psi}$ satisfies

$$\nabla^2 \bar{\psi} = 0 \qquad (19)$$

Equation (18), when written in terms of the vector components, constitutes a set of three equations to be solved for ψ_x, ψ_y, and ψ_z. They are:

$$\nabla^2 \psi_x = 0$$

$$\nabla^2 \psi_y = 0 \qquad (20)$$

$$\nabla^2 \psi_z = 0$$

Equations (20) are similar to equation (14). But the boundary conditions are not as simple as (15) and body shape needs to be specified to formulate them. Due to the complexity involved in the boundary conditions and solution procedures the flow model covers only the two dimensional and axisymmetrical cases.

The two-dimensional equation (14) for ψ is useful where the sampling inlet is either axisymmetrical or when the ratio of height to width of the inlet is large enough to discard one dimension. Even if these conditions are not satisfied for a particular inlet, equation (14) will still approximate the flow in the core region of the sampler inlet.

Formulation of the Problem for Specific Inlet Geometries

Parallel Plate Inlet--

This type of inlet is often encountered in the horizontal elutriator. Figure 7 shows the nomenclature for the fluid flow region. As can be seen from the figure, some transverse distance Y_0 from the centerline of the probe defined where the fluid stream line maintains a flow unperturbed by the sampler. This assumption is necessary because a numerical solution for the problem can be obtained only for a finite number of mesh points.

The required boundary conditions now have to be specified at the centerline, the probe surface, the constant ordinate Y_0 and section I and II as shown in Figure 7.

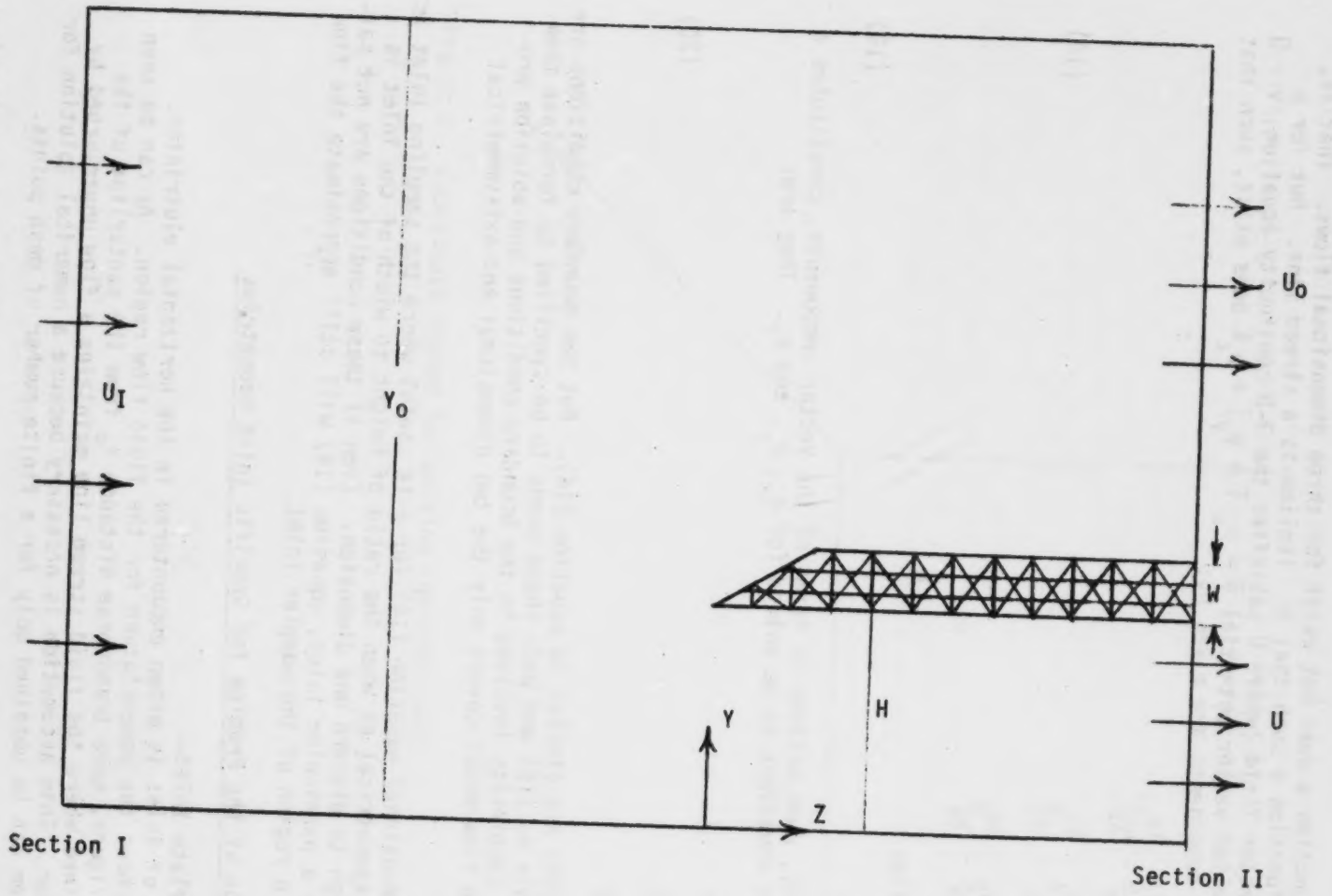


Figure 7. Nomenclature for flow region--two dimensional case.

When Section I and II are sufficiently far up and downstream from the inlet and the disturbed flow region, the velocities at these stations can be considered uniform and axial only. The axial component of velocity U_z and transverse component U_y are related to the stream as follows:

$$U_z = \frac{\partial \psi}{\partial y} \tag{21}$$

$$U_y = -\frac{\partial \psi}{\partial z} \tag{22}$$

The expression for ψ at sections I and II can be obtained by integrating equation (21):

$$\psi = U_y + C_1 \tag{23}$$

Where U is the appropriate velocity of the fluid and C_1 is an orbitrary comtant of integration for which a value of zero can be assigned.

Then on the centerline of the probe (upstream and inside the probe) the value of the stream function becomes:

$$\psi \, \text{¢} = \psi \text{ at centerline} = 0 \tag{24}$$

By virtue of equation (23), the values for ψ at other boundaries follow:

$$\psi \text{ at Section I} = \psi_I = U_I y$$
$$\psi \text{ at } y_o = \psi y_o = U_I y_o \tag{25}$$

y at Section II is given in by three equation valid over various regions.

$$\psi_{IP} = \psi \text{ at Section II inside the probe}$$
$$= U_y \qquad 0 \leq y \leq H \tag{26}$$
$$\psi_{OP} = \psi \text{ at Section II outside the probe}$$
$$= U_o y \qquad (H+W) \leq y \leq y_o$$

By use of mass balance at Sections I and II, expression for ψ_{OP} becomes (Appendix A)

$$\psi_{OP} = \frac{(U_I y_o - UH)}{[y_o - (H+W)]} * [y - (H+W)] + UH$$
$$\text{for } (H+W) \leq y \leq y_o \tag{27}$$

$$\psi_{PS} = \psi \text{ on the probe surface} = UH \text{ for } H \leq y \leq (H+W) \tag{28}$$

Divide the velocities by U_I and the distances by H to make them nondimensional. Denoting the nondimensional quantities by stars, the boundary conditions become:

$$\psi_I^* = y^*$$

$$\psi_{IP}^* = U^* y^*$$

$$\psi_{OP}^* = \frac{(\psi_o^* - U^*)}{(\psi_o^* - (H+W^*))} * (y^* - (1+W^*)) + U^* \qquad (29)$$

$$\psi_{CL}^* = 0$$

$$\psi^*_{PS} = U^*$$

Equation (14) together with the boundary conditions (29) pose a well defined problem.

Circular Tube--
Circular inlets are the most commonly used geometry in aerosol sampling. Although circular tubes are very rarely found in the personal samplers for aerosols, the inlets to the closed face filters can be approximated by the circular tube geometry. Open faced filters can be regarded as a wide and very short circular tubes terminating in a filter.

Figure 8 gives the flow field boundary description and nomenclature. A circular tube of radius R, and wall thickness W, is considered to be located at Z = 0. the axial and radial velocity components, U_z and U_r are given as follows:

$$U_z = \frac{1}{r} \frac{\partial \psi}{\partial r}$$
$$U_r = \frac{-1}{r} \frac{\partial \psi}{\partial z} \qquad (30)$$

Following similar procedure used for parallel plates,

$$\psi_I = \tfrac{1}{2} U_I r^2$$

$$\psi_{IP} = \tfrac{1}{2} U r^2$$

$$\psi_{OP} = \tfrac{1}{2} \frac{(U_I R_o^2 - UR^2)}{(R_o^2 - (R+W)^2)} \cdot (r^2 - (R+W)^2) + \tfrac{1}{2} UR^2 \qquad (31)$$

$$\psi_{\text{CL}} = \psi \text{ at centerline} = 0$$

Nondimensionalizing velocities by $\frac{1}{2} U_I$ and distances by R boundary conditions become:

$$\psi_I^* = r^{*2} \qquad 0 \le r^* \le R_o^*$$

$$\psi_{IP}^* = U^* r^{*2} \qquad 0 \le r^* \le 1 \qquad (32)$$

$$\psi_{OP}^* = \frac{(R_o^{*2} - U^*)}{(R_o^{*2} - (1+W^*)^2)} (r^{*2} - (1+W^*)^2) + U^*$$
$$(1+W^*) \le r \le R_o^*$$

$$\psi^*_{\text{CL}} = 0$$

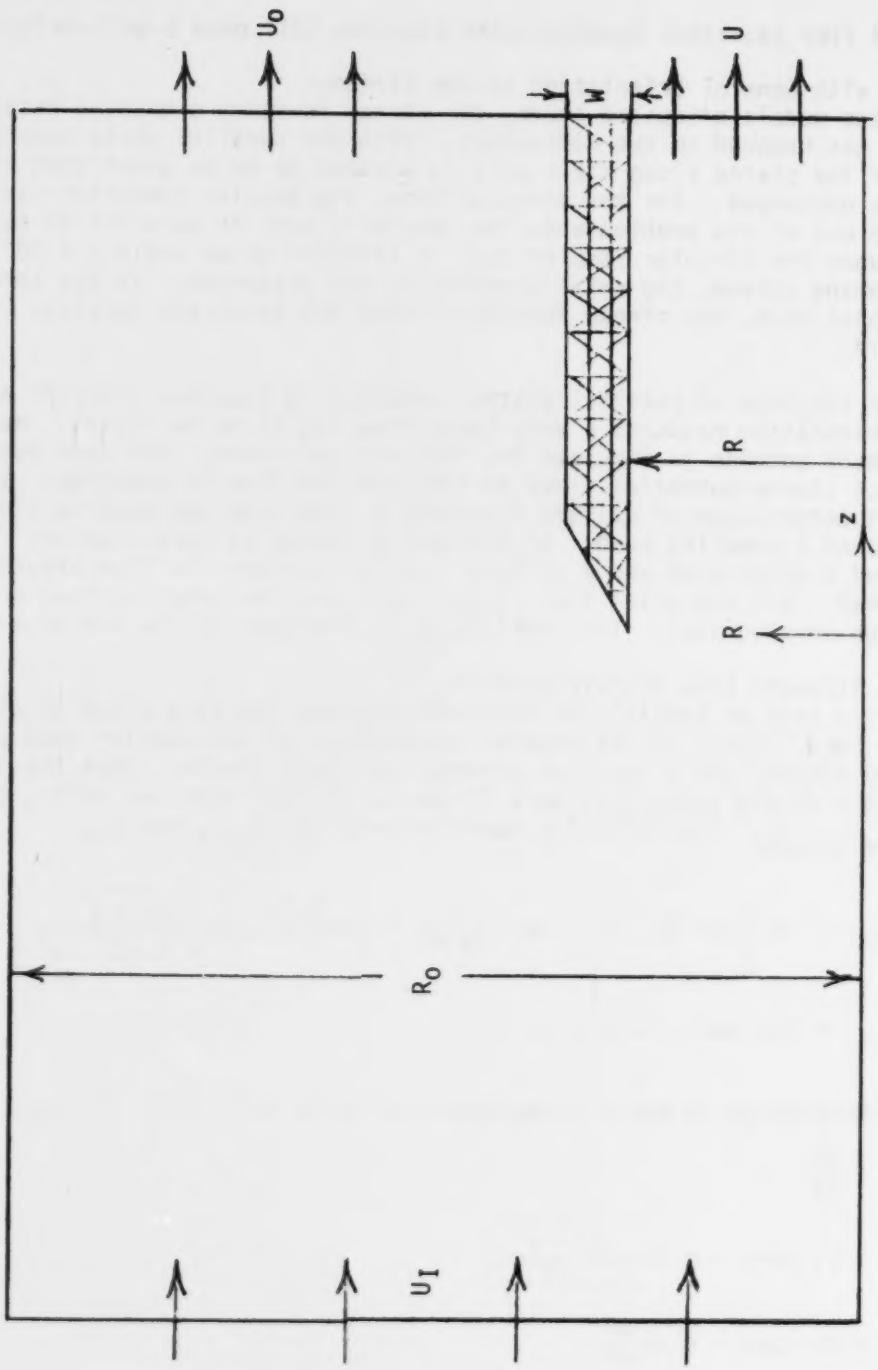


Figure 8. Nomenclature for flow region for circular inlet.

Inviscid flow equations together with Equation (31) pose a well-defined problem.

Sampler with General Orientation to the Stream--
In the two models discussed above, the stream function ψ existed because the problem was reduced to two dimensions. With the parallel plate sampler, the width of the plates along the x-axis is assumed to be so great that the flow field is unchanged. For the circular tube, the angular dimension can also be factored out of the problem when the sampler's axis is parallel to the initial flow. When the circular sampler face is oriented at an angle $\alpha \neq 90°$ towards the oncoming stream, the axial symmetry is not preserved. In the three-dimensional case, the stream function ψ does not exist and Equation (19) is not valid.

Even for the case of parallel plates, solution to Equation (14) for an arbitrary orientation requires a very large flow region to be solved. Hence other approximate methods to simulate the flow are necessary. One such method is to superpose simple potential flows to simulate the flow in question. In this study, superposition of uniform flow over a line sink was used to simulate the flow around a sampling head. In the past a number of investigators[7,12,13,15,16] have used a point sink and a uniform flow to simulate the flow around the sampling head. But the point sink is isotropic and the sampling head orientation cannot be incorporated. This difficulty is overcome by the use of a line sink.

Uniform Strength Sink Distribution--
Let a line sink of length d be distributed along the face plane OA of the sampler head. Let α be the angular orientation of the sampler head with the oncoming stream, and m the sink strength per unit length. Then the stream function ψ at any point P (Figure 9) can be divided into two parts, ψ_{sink} and $\psi_{uniform\ stream}$. The following equation provides ψ_{sink} and ψ_{us}.

$$\psi_{sink} = -m \left[\tan^{-1}\frac{y}{x} \cdot x - \tan^{-1}\frac{y}{(x-d)} \cdot (x-d) + y \ln \frac{\sqrt{x^2+y^2}}{\sqrt{(x-d)^2+y^2}} \right] \tag{33}$$

$$\psi_{u.s.} = U_I y \cos\alpha + U_I x \sin\alpha \tag{34}$$

U_x = velocity in the x direction

$$= \frac{\partial \psi}{\partial y}$$

$$= U_I \cos\alpha - m \ln \frac{\sqrt{x^2+y^2}}{\sqrt{(x-d)^2+y^2}} \tag{35}$$

$$= U_I \cos\alpha - m \ln \frac{OP}{AP}$$

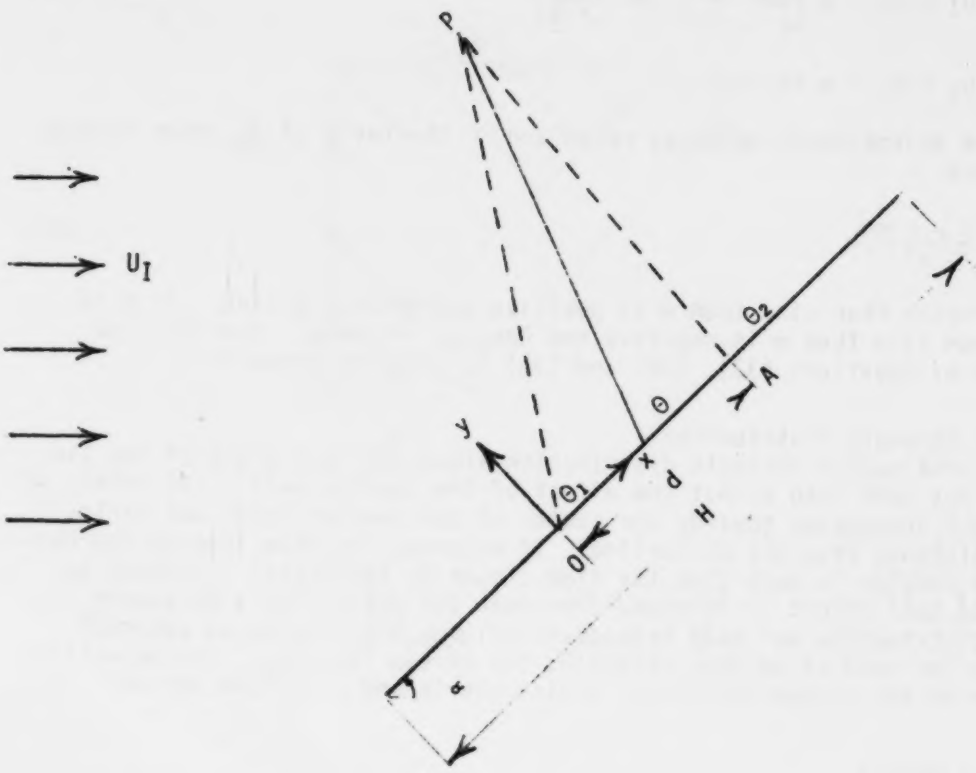


Figure 9. Line sink distribution along face plane of the sampler.

U_y = velocity in y direction

$$= -\frac{\partial \psi}{\partial x}$$

$$= U_I \sin\alpha + m\left[\tan^{-1}\frac{y}{x} - \tan^{-1}\frac{y}{x-d}\right] \quad (36)$$

$$= U_I \sin\alpha + m(\theta_1 - \theta_2)$$

If U is the anisokinetic velocity ratio and f the ratio of $\frac{d}{2H}$, then through mass balance

$$m = \frac{U - \sin\alpha}{\pi f} \quad (37)$$

If U is greater than $\sin\alpha$ then m is positive and denotes a sink. If U is smaller than $\sin\alpha$ then m is negative and denotes a source. The detailed derivation of Equations (33), (38) and (36) is given in Appendix B.

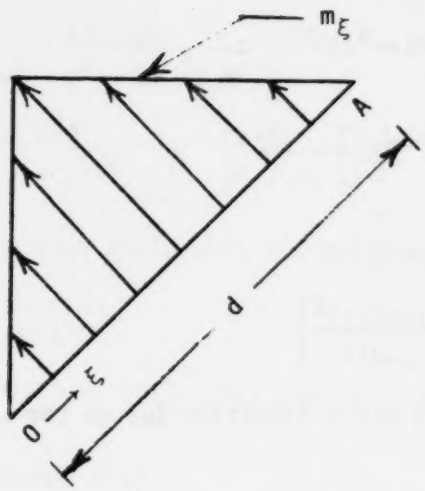
Triangular Strength Distribution--
A uniform sink/source strength distribution along the face plane of the sampler does not take into accout the effect of the sampler wall. The effect of sampler wall propagates towards the center of the sampler inlet and varies with the distance from the centerline. In essence, the flow through the center of the sampler is more than the flow closer to the walls. In order to approximate this effect of retarded flow near the walls, the sink/source strength distribution was made triangular (Figure 10). Using an approach similar to the case of uniform strength, the stream function can be written as the sum of two stream functions, ψ sink/source and ψ uniform stream.

Then from Figure 9

$$S = \int_0^d m_\xi \cdot \theta \cdot d\xi \quad (38)$$

Where
$$m_\xi = m\xi; \qquad 0 \leq \xi \leq d/2$$
$$= m(d-\xi); \quad d/2 \leq \xi \leq d \quad (39)$$

and $$\theta = \cot^{-1}\frac{(x-\xi)}{\bar{w}} \quad (40)$$



$$m_\xi = m\xi \qquad 0 \leq \xi \leq d/2$$
$$= m(d-\xi) \qquad d/2 \leq \xi \leq d$$

Figure 10. Triangular source/sink strength distribution.

The details of integration of equation (38) with equation (39) and (40) are given in Appendix C. The stream function due to source/sink ΨS is given by

$$\Psi S = \text{Cot}^{-1}\, x/y \left[mx^2/2 - my^2/2\right]$$

$$+ \text{Cot}^{-1}\, \frac{x-d/2}{y} \left[my^2 - m(x-d/2)^2\right]$$

$$+ \text{Cot}^{-1}\, \frac{x-d}{y} \left[\frac{m(d-x)^2}{2} - \frac{my^2}{2}\right]$$

$$+ myx/2\, \ell n\left[\frac{y^2+x^2}{y^2+(x-d/2)^2}\right]$$

$$+ my\,(d-x)/2\, \ell n\left[\frac{y^2+(x-d/2)^2}{y^2+(x-d)^2}\right] \qquad (41)$$

The velocities U_x and U_y in x and y direction due to the sink are given as follows.

$$Ux = \frac{\partial \Psi}{\partial y}$$

$$= mx/2\, \frac{x^2-y^2}{x^2+y^2} - my\, \cot^{-1}\, x/y$$

$$+ m\,(x-d/2)\, \frac{y^2-(x-d/2)^2}{y^2+)x=d/2)^2} + 2my\, \cot^{-1}\, \frac{x-d/2}{y}$$

$$+ \frac{m\,(x-d)}{2}\ \ \frac{(d-x)^2-y^2}{(d-x)^2+y^2} - my\, \cot^{-1}\, \frac{x-d}{y}$$

$$+ \frac{mx}{2}\, \ell n\left[\frac{y^2+x^2}{y^2+\left(\frac{x-d}{2}\right)^2}\right] + my\, x\left[\frac{1}{x^2+y^2} - \frac{1}{y^2+\left(x-\frac{d}{2}\right)^2}\right]$$

$$+ m(d-x)\, \ell n\left[\frac{y^2+(x-d/2)^2}{y^2+(x-d)^2}\right] + my^2\,(d-x)\left[\frac{1}{y^2+(x-d/2)^2} - \frac{1}{y^2+(x-d)^2}\right] \qquad (42)$$

and $Uy = -\frac{\partial \Psi}{\partial x}$

$$= my/2\, \frac{x^2-y^2}{x^2+y^2} - mx\, \cot^{-1}\, x/y$$

$$+ my\ \ \frac{y^2-(x-d/2)^2}{y\ +(x-d/2)^2} + 2m\,(x-d/2)\, \cot^{-1}\, \frac{x-d/2}{y}$$

$$+ my/2\, \frac{(d-x)^2-y^2}{(d-x)^2+y^2} + m\,(d-x)\, \cot^{-1}\, \frac{x-d}{y}$$

$$+ my/_2 \ln\left[\frac{y^2 + x^2}{y + (x-d/2)^2}\right] + my/_2 \ln\left[\frac{y^2 + (x-d/2)^2}{y^2 + (x-d)^2}\right]$$

$$- myx\left[\frac{x}{y^2 + x^2} - \frac{(x-d/2)}{y^2 + (x-d/_2)^2}\right]$$

$$- my\,(d-x)\left[\frac{x-d/2)}{y^2 + (x-d/_2)^2} - \frac{(x-d)}{y^2 + (x-d)^2}\right] \qquad (43)$$

The velocities in the flow field with the uniform stream will be

$$Ux = Ux + Uo\cos\alpha$$
$$Uy = Uy + Uo\sin\alpha \qquad (44)$$

The value of m in equations (41), (42) and (43) can be obtained by mass balance as per Appendix C.

$$m = -2(U-\sin\alpha)/\pi f^2 \qquad (45)$$

where

U is the anisokinetic velocity ratio.
α the angular orientation of the sampler head.
f the fraction of the diameter over which the sink/source is assumed to be distributed.

The equation (41) - (45) define the flow field completely. This simulation of flow by superposition accounts for the angular orientation and the anisokinetic sampling. The effect of inlet geometry is not taken into account.

PARTICLE MOTION

AEROSOL MECHANICS OF RESPIRABLE PARTICLES

The general theory of the dynamics of spherical bodies suspended in a fluid as a continuum restricts analysis to the condition where the Knudsen number of the body, Kn, approaches zero. In practice, these results can be applied to particle behavior under conditions of Kn >> 1, then the continuum theory cannot be applied and free molecular theory takes over. For intermediate Knudsen number particle dynamics, the continuum theory needs to be supplemented with a slip correction factor. All the above theories have been well documented in the literature.[13,14]

The various forces acting on a particle are as follows:

- inertial
- gravitational
- diffusional
- electrostatic

Inertial

In the course of movement, particles of aerosol may acquire motion relative to the suspending gas, due to their inability to conform to the fluid flow instantaneously. There is a certain amount of time lag in which the particle is not affected by local velocity changes in the flow. This is characterized by τ, the relaxation time of the particles. τ is defined as the ratio of particle terminal settling velocity to the acceleration due to gravity. In the same manner, if an aerosol at rest is accelerated to a velocity V/e in time τ. Hence, in a velocity gradient field, the particle overshoots the fluid when decelerating or it undershoots it when accelerating.

Stokes number, K (the ratio of the stop distance of the particle to the characteristic length of the system), is used to indicate the importance of inertial effects for a given set of conditions. The smaller the value of K, the more negligible the inertia:

$$K = \frac{U_0 \tau}{L} = \frac{\rho d^2 U_0}{18 \eta L} \qquad (45)$$

where U_o = the free stream velocity

τ = the relaxation time of particles

L = characteristic length (cm)

ρ = the density of the aerosol particle (gm/cc)

d = diameter of the particle (cm), and

η = the viscosity of the medium (gm cm/sec).

Gravitational

Gravitational force on particles is given by mg, where m is the mass of the particles and g is the acceleration due to gravity. The terminal settling velocity, V_s, is given by

$$V_s = \tau g \tag{47}$$

where

τ = the relaxation time of particles, and

g = acceleration due to gravity (cm/sec).

The importance of gravitational effect is indicated by the Froude number (Fr):

$$Fr = \frac{V_s^2}{Lg} = \frac{V_s \tau}{L} \tag{48}$$

Diffusional

Diffusion is the most dominant force on small particles (d<.2 μm). Particles not under the influence of external forces diffuse in a random fashion called Brownian motion. Diffusion also occurs because of velocity gradients, concentration gradients, and thermal gradients.

The characteristic numbers are the Schmidt number, Sc, and the Peclet number, Pe.

$$Sc = \frac{\gamma}{\varepsilon} \tag{49}$$

where

γ = kinematic viscosity (cm /sec), and

ε = diffusivity of particles (cm /sec).

The Schmidt number indicates the ratio of momentum transfer to mass transfer. Higher Sc values mean Brownian Diffusion is not as important as convective diffusion. The combined effects of diffusion and fluid motion (convection) on particle transport can be expressed as a function of the Peclet number, Pe:

$$Pe = \frac{V_o D}{\varepsilon} \tag{50}$$

where

V_o = the initial inviscid velocity (cm/sec), and

D = the diameter of the sampler inlet (cm).

In this study diffusion was not included.

Electrostatic Force

The electrostatic force considered in the model is only the coulombic force between point charges of magnitude Qp and Qc located at the center of the particle and the collector respectively. The coulombic force between a charged particle and sampler head is given by

$$Fc = - \frac{Qc\ Qp}{4\pi\varepsilon_o R^2} \tag{51}$$

where

Qc = charge on the collector/unit length

Qp = charge on the particle

ε_o = dielectric constant of air, and

R = distance between the center of particle to the surface of collector.

If the linear dimension of the collector surface is L, the electrostatic force can be obtained by integrating equation (51) over the length of the collector. Using the coordinate system shown in Figure 11, the total force on the particle can be written as follows

$$F = \int_o^L Fc\,dx \tag{52}$$

Fx = x component of the force

$$= \frac{K}{(y-a)} (\text{Sin}\ \theta_1 - \text{Sin}\ \theta_2) \tag{53}$$

and

Fy = y component of the force

$$= \frac{K}{(y-a)} (\text{Cos}\ \theta_2 - \text{Cos}\ \theta_1)$$

where

$$K = \frac{Qc\ Qp}{4\pi\varepsilon_o}$$

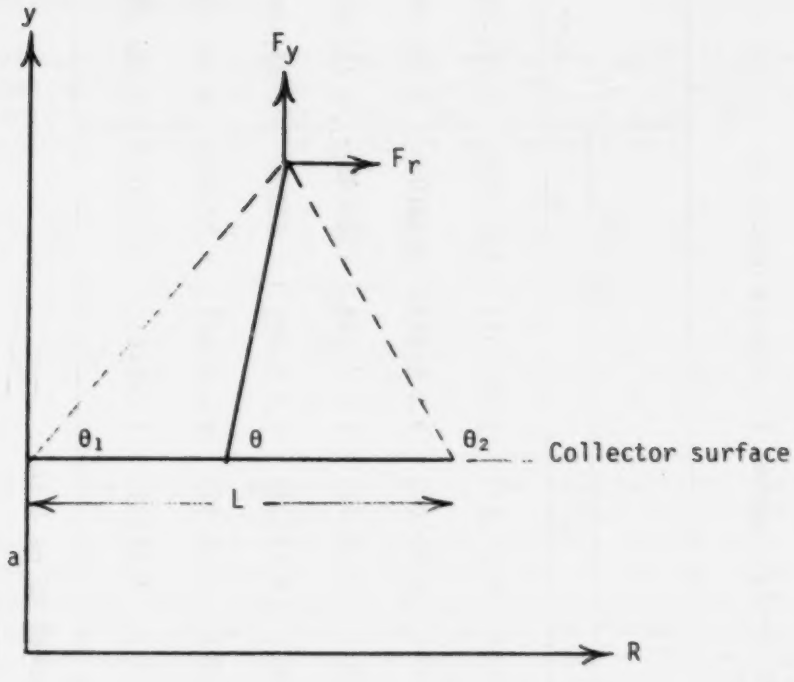


Figure 11. Coordinate system for electrostatic force calculation.

Equation (51) is valid as long as the particle and the collector are not very close. For closer distances, the equations (52) and (53) become infinite and image forces have to be taken into account. This study does not take into account the image forces.

Table 2 provides values of non-dimensional parameters such as Stokes number K, Peclect number Pe, and Froude number Fr for various particles.

Table 2. Typical values of Stokes number K, Peclet number Pe, Froude number Fr for unit density spheres.*

Particle Radius (μ) \ Flow Velocity (cm/sec)	10		50		100		
	K	Pe	K	Pe	K	Pe	Fr**
0.25	1.02(-05)	1.6 (07)	5.1 (-05)	8.0 (07)	1.02(-04)	1.6 (08)	1.03(-09)
0.50	3.6 (-05)	3.65(07)	1.8 (-04)	1.83(08)	3.6 (-04)	3.65(08)	1.25(-08)
1.00	1.3 (-04)	7.85(07)	6.5 (-04)	3.93(08)	1.3 (-03)	7.85(08)	1.73(-07)
2.00	5.1 (-04)	1.63(08)	2.55(-03)	8.15(08)	5.1 (-03)	1.63(09)	2.56(-06)
5.00	3.1 (-03)	4.17(08)	1.55(-02)	2.09(09)	3.1 (-02)	4.17(09)	9.55(-05)
10.00	1.24(-02)	8.43(08)	6.2 (-02)	4.22(09)	1.24(-01)	8.43(09)	1.50(-03)

* Characteristic dimension 'L' of the sampler is assumed to be 1 cm.

** Froude number 'Fr' is independent of flow velocities.

EQUATION OF MOTION OF PARTICLES

When an aerosol particle travels in a moving medium the particle generally tends to lag behind the flow of fluid. Assuming that the Stokes relation for the particle drag may be used, then within the continuum approximation of the fluid the equation of motion of the particle may be written as:[14]

$$\frac{\pi}{6} d^3 \rho \frac{d\bar{v}}{dt} = 3\pi\eta d (\bar{U}-\bar{v}) + \frac{\pi}{6} d^3 \rho_g \frac{d\bar{U}}{dt} + \frac{\pi}{12} d^3 \rho_g \left(\frac{d\bar{U}}{dt} - \frac{d\bar{v}}{dt}\right)$$
$$+ \frac{3}{2} d^2 \sqrt{\pi \rho_g \eta} \int_{t_o}^{t} \frac{dt^1 \left(\frac{dU}{dt^1} - \frac{dv}{dt^1}\right)}{\left(t - t^1\right)^{\frac{1}{2}}} + F_e \qquad (54)$$

The first term on the left in equation (54) is the resultant force acting on the particle. The first term on the right is the viscous resistance given by the Stokes law. The second term is due to the pressure gradient in the gas surrounding the particle, caused by the acceleration of the particle. The third term denotes the force required to accelerate the apparent mass of the particle relative to the ambient gas. The fourth term, known as the Basset term, accounts for the deviation from the steady state in the gas flow pattern. The last term is the force resulting from external potential. In general, the second, third and fourth terms will be negligible, so equation (54) in simplified form will be

$$\frac{\pi}{6} d^3 \rho \frac{d\bar{v}}{dt} = 3\pi\eta d (\bar{U} - \bar{v}) + \bar{F}_e \qquad (55)$$

or

$$\frac{d\bar{v}}{dt} = \frac{\bar{U} - \bar{v}}{\tau} + \bar{F}_{em} \qquad (56)$$

where $\bar{F}_{em}$ is the external force per unit mass. Divide velocities by U_I and time by L/U_I to obtain nondimensional form of equation (56). Denoting the nondimensional quantities by stars, the equation of motion becomes:

$$\frac{d\bar{V}^*}{dt^*} = \frac{\bar{U}^* - \bar{V}^*}{K} + \bar{F}_{em}^* \qquad (57)$$

where

$$K = \text{Stokes number of particle} = \frac{U_I \tau}{L}$$

$$\bar{F}_{em} = \frac{\bar{F}_{em} \cdot L}{U_I^2} \qquad (58)$$

METHODS OF SOLUTION

In order to estimate the sampling error, it is necessary to solve the equation of particle motion (equation 48) together with the fluid flow equations. Analytical solutions to partial differential equations such as equations (57) or (14) can be obtained for only very simple boundary configurations. Approximate solutions, however, can be obtained by numerical methods by solving the finite differenced equations of the governing differential equation.

FLUID FLOW

As indicated in the previous section, the model for the fluid flow has two options. The first option is to solve the flow equation (19) with proper boundary conditions to obtain the numerical solution to the exact problem. The second option is to use the superposition of simple flows to approximate the actual flow conditions. The first option requires numerical solution of equation (14) and can be used only when the sampling head is facing the oncoming stream. For general orientation of the sampling head, the second option is used and requires numerical evaluation of the flow velocities given by equations (41)-(45) for use in determining the trajectory of particles.

Sampler Facing the Stream

Thin-walled Plates--
Let the flow field be divided into a grid as shown in Figure 12. Then, Laplace's equation (14) can be written in finite difference form and solved by iteration.

Each iteration of the finite difference equation is analogous to solving the time-dependent version of equation (14):

$$\frac{\partial \psi}{\partial \tau} = \nabla^2 \psi \tag{59}$$

We are not interested in the physical significance of this transient solution, but a step in time Δt in the time-dependent ψ is a convenient representation for an iteration of the time-independent function. As the solution to equation (59) approaches steady state, we have also converged to the desired solution of Laplace's equation (14).

Now, we write equation (59) in discrete form for point I, J using FTCS (Forward Time Centered Space) differencing

$$\frac{\psi_{IJ}^{k+1} - \psi_{IJ}^{k}}{\Delta t} = \frac{\delta^2 \psi}{\Delta x^2} + \frac{\delta^2 \psi}{\Delta y^2} \tag{60}$$

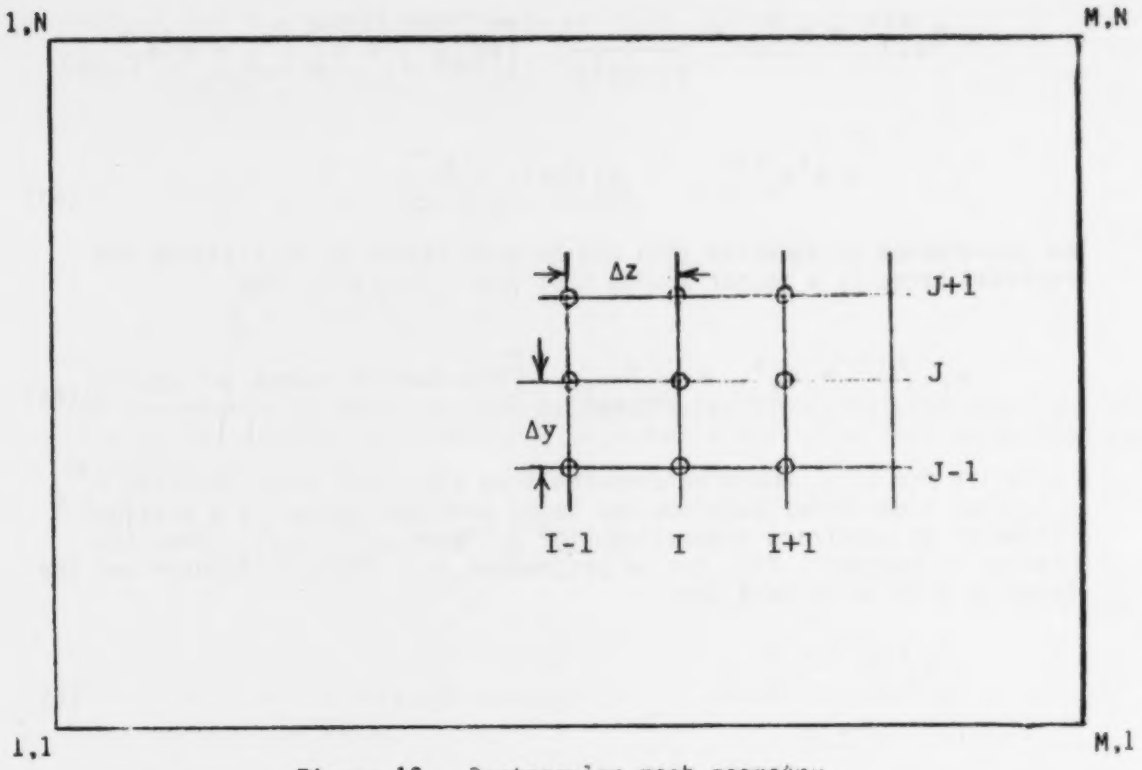


Figure 12. Rectangular mesh geometry.

where

$\psi^{k}_{I,J}$ = the stream junction ψ at (I,J) at the k^{th} time step.

$$\frac{\delta^2\psi^k}{\Delta z^2} = \frac{\psi^k_{I-1,J} - 2\,\psi^k_{I,J} + \psi_{I+1,J}}{\Delta z^2} \qquad (61)$$

$$\frac{\delta^2\psi^k}{\Delta y^2} = \frac{\psi^k_{I,J-1} - 2\,\psi^k_{I,J} + \psi_{I,J+1}}{\Delta y^2}$$

For the difference equation (60) to be stable the condition is:

$$\frac{\Delta t}{\Delta z^2} + \frac{\Delta t}{\Delta y^2} \leq 1/2 \qquad (62)$$

Since we wish to approach the solution as rapidly as possible, we take the largest possible Δt from equation (62).

Defining the mesh aspect $\beta = \Delta z/\Delta y$, then:

$$\psi_{I,J}^{k+1} = \psi_{I,J}^{k} + \frac{1}{2(1+\beta^2)} \left[\psi_{I+1,J}^{k} + \psi_{I-1,J}^{k+1} + \beta^2 \psi_{I,J+1}^{k}\right.$$

$$\left. + \beta^2 \psi_{I,J-1}^{k+1} - 2(1+\beta^2)\ \psi_{I,J}^{k}\right] \tag{63}$$

The convergence of equation (63) can be made faster by multiplying the bracketed terms by a factor ω such that $1 \leq \omega < 2$ [28,29]. Then

$$\psi_{I,J}^{k+1} = \psi_{I,J}^{k} + \frac{\omega}{2(1+\beta^2)} \left[\text{Bracketed terms as above}\right] \tag{64}$$

A solution for $\psi_{I,J}^{k+1}$ would be computed from the lower order solution $\psi_{I,J}^{k}$. An initial lower order solution has to be provided either by a previous problem or by arbitrary assumptions for ψ. When $\psi_{I,J}^{k+1} \rightarrow \psi_{I,J}^{k}$, then the solution is reached. This can be programmed in a digital computer and the iteration will be stopped when

$$\frac{\left|\psi_{I,J}^{k+1} - \psi_{I,J}^{k}\right|}{\psi_{I,J}^{k}} \leq \xi \ \text{(error limit)} \tag{65}$$

The velocities are calculated as follows.

$$\begin{aligned} U_z(I,J) &= (\psi(I,J+1) - \psi(I,J-1))/2\cdot\Delta y \\ U_y(I,J) &= -[\psi(J+1,J) - \psi(J-1,J)]/2\cdot\Delta z \end{aligned} \tag{66}$$

Thin-walled Tube--
The time dependent problem for circular tube is given by

$$\frac{\partial \psi}{\partial t} = \frac{\partial^2 \psi}{\partial z^2} - \frac{1}{r}\ \frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial r^2} \tag{67}$$

Writing equation (67) in discreetized form for point I,J using FTCS differencing

$$\frac{\psi_{IJ}^{k+1} - \psi_{IJ}^{k}}{\Delta t} = \frac{\delta^2 \psi^{k}}{\Delta z^2} + \frac{\delta^2 \psi^{k}}{\Delta r^2} - \frac{1}{R}\frac{\delta \psi^{k}}{\Delta r} \tag{68}$$

where

ψ_{IJ}^{k} = stream function at (I,J) at k^{th} time step

R = Radial Coordinate at (I,J)

$$\frac{\delta^2\psi^k}{\Delta z^2} = \frac{\psi_{I-1,J} - 2\psi^k_{I,J} + \psi^k_{I+1,J}}{\Delta z^2}$$

$$\frac{\delta^2\psi^k}{\Delta r^2} = \frac{\psi^k_{I,J-1} - 2\psi^k_{I,J} + \psi^k_{I,J+1}}{\Delta r^2}$$

$$\frac{\delta\psi^k}{\Delta r} = \frac{\psi^k_{I,J+1} - \psi^k_{I,J-1}}{2\cdot\Delta r}$$

The convergence of equation (68) is made faster by multiplying the right hand side of the equation by a factor w such that $1 \leq w \leq 2$. The flow velocities are calculated as follows

$$U_z(I,J) = \frac{1}{R} \frac{\psi_{I,J+1} - \psi_{I,J-1}}{2\cdot\Delta r}$$

$$U_r(I,J) = -\frac{1}{R} \frac{\psi_{I+1,J} - \psi_{I-1,J}}{2\cdot\Delta z} \qquad (69)$$

Thick-Walled Tube/Plate

Depending on the shape and thickness of the sampler wall the grid size in any one direction may not be equal throughout the flow region (Figure 13).

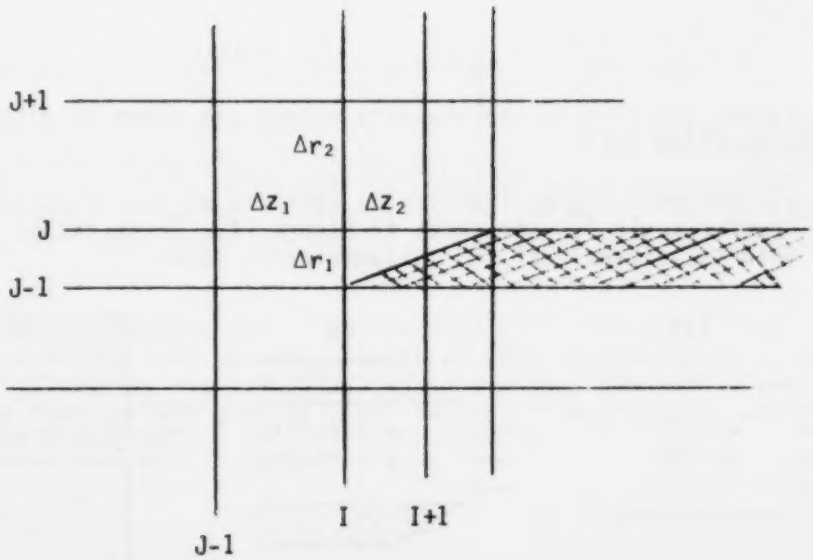


Figure 13. Unequal grid size.

Writing the governing equation (67) in discreetized form for unequal grid size and grouping the terms gives:

$$\psi_{I,J}\left\{\frac{2}{(\Delta z_1\cdot\Delta z_2)} - \frac{1}{R}\left[\frac{\Delta r_1-\Delta r_2}{\Delta r_1\cdot\Delta r_2}\right] + \frac{2}{\Delta r_1\cdot\Delta r_2}\right\}$$

$$= \psi_{I-1,J}\left\{\frac{2}{(\Delta z_1+\Delta z_2)\cdot\Delta z}\right\} + \psi_{I+1,J}\left\{\frac{2}{(\Delta z_1+\Delta z_2)\cdot\Delta z_2}\right\}$$

$$+ \psi_{I,J-1}\left\{\frac{1}{R}\cdot\frac{\Delta r_2}{(\Delta r_1+\Delta r_2)\cdot\Delta r_1} + \frac{2}{(\Delta r_1+\Delta r_2)\cdot\Delta r_1}\right\}$$

$$+ \psi_{I,J+1}\left\{-\frac{1}{R}\cdot\frac{\Delta r_1}{(\Delta r_1+\Delta r_2)\cdot\Delta r_2} + \frac{2}{(\Delta r_1+\Delta r_2)\cdot\Delta r_2}\right\} \qquad (70)$$

for circular tube,

$$\psi_{I,J}\left\{\frac{2}{\Delta z_1\cdot\Delta z_2} + \frac{2}{\Delta r_1\cdot\Delta r_2}\right\} = \psi_{I-1,J}\left\{\frac{2}{(\Delta z_1+\Delta z_2)\cdot\Delta z_1}\right\}$$

$$+ \psi_{I+1,J}\left\{\frac{2}{(\Delta z_1+\Delta z_2)\cdot\Delta z_2}\right\}$$

$$+ \psi_{I,J-1}\left\{\frac{2}{(\Delta r_1+\Delta r_2)\cdot\Delta r_1}\right\}$$

$$+ \psi_{I,J+1}\left\{\frac{2}{(\Delta r_1+\Delta r_2)\cdot\Delta r_2}\right\} \qquad (71)$$

for plates

The stream function at the boundary points are given by either equation (29) or by equation (32).

The various wall shapes that can be realized by use of wall thickness 'w' and chamfer angle 'α' are shown in Figure 14. These shapes can be obtained for both two-dimensional and axisymmetrical cases.

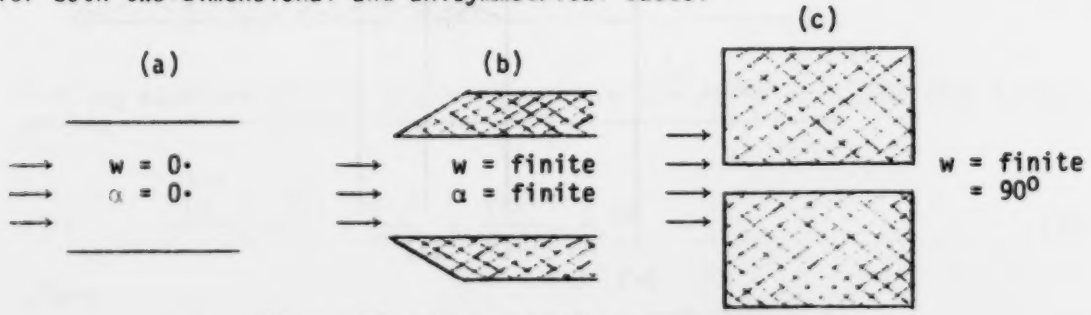


Figure 14. Various inlet geometries.

Sampler with General Orientation

When the sampler is oriented at an angle 'α' to the on-coming stream, the approximate solution developed earlier is used. Equations (41)-(45) will be evaluated for use in particle trajectory calculation.

PARTICLE MOTION

The equation of motion of particles as given by equation (48) is solved by calculating the trajectories of particles. The method that is used to do this is the predictor-corrector method together with an iterative convergence. Even though the predictor-corrector method is a standard one and can be found in any numerical methods book, we will give a short description of the method here.

Prediction

If the particle is at position $\vec{x}=\vec{x}o$ at time t=0, then at t=t+Δt, the particle is predicted to be at

$$\vec{x}_p = \vec{x}_o + \Delta t \cdot \vec{v} \quad \Big| \quad \vec{x} = \vec{x}_o \tag{72}$$

Correction

The above equation assumes that the velocity, v, of the particle remains constant within the step. But actually it is a changing function:

$$\vec{x}_c = \vec{x}_o + \Delta t \cdot v \quad \Big| \quad \vec{x} = \frac{\vec{x}_o + \vec{x}_p}{2} \tag{73}$$

Equation (73) is iterated until the corrected value $\vec{x}_c$ converges.

The above procedure is continued until the particle either touches the wall of the sampling probe, in which case it is captured and lost from the sample, or enters the probe inlet, in which case it could either be transported to the sensing instrument or be lost by deposition to the walls. A limiting trajectory which will just graze the inside of the probe wall can then be calculated.

In order to calculate the velocity v for equation (73), the simplified equation for particle motion (48) is used. Equation (48) is the result of the various forces influencing the particle. In this study, the particle motion is determined by inertial and gravitational forces. Electrostatic and diffusion forces are neglected.

Assumptions

The following assumptions are used in obtaining a solution to the particle motion equation.

- The particles are uniformly distributed and, at a large distance upstream from the probe inlet, move with the same velocity as the free stream fluid.
- The particles are spherical and do not change in size due to agglomeration, evaporation or condensation.
- The particles are considered to be sufficiently small in comparison with probe size and they move as individual particles with no hydro-dynamic interactions among themselves or between themselves and the probe walls.

Solution Procedure

At a large distance upstream (5 radii) from the probe inlet the fluid flow is uniform and the particles move with the stream. As the flow approaches the sampling probe the disturbance due to the presence of the probe is felt and the particles due to their inertia are not able to follow the fluid flow.

The equation (48) can be written in terms of the velocity components as

$$\frac{dV_x}{dt} = \frac{U_x - V_x + V_{ext-x}}{K}$$
$$\frac{dV_y}{dt} = \frac{U_y - V_y + V_{ext-y}}{K} \qquad (74)$$

Now let the particle be at position (x_o, y_o) at time $t=t_o$. Then at time $t=t_o+\Delta t$, $V_x=V_xP$, $V_y=V_yP$ are the predicted values of velocities.

$$V_xP = V_x + \left.\frac{dV_x}{dt}\right|_{\substack{x=x_o \\ y=y_o}} \cdot \Delta t$$
$$V_yP = V_y + \left.\frac{dV_y}{dt}\right|_{\substack{x=x_o \\ y=y_o}} \cdot \Delta t \qquad (75)$$
$$xP = x_o + V_xP \cdot \Delta t$$
$$yP = y_o + V_yP \cdot \Delta t$$

The corrected values are

$$V_xC = V_x + \left.\frac{dV_x}{dt}\right|_{\substack{x = \frac{x_o+x_p}{2} \\ y = \frac{y_o+y_p}{2}}} \cdot \Delta t$$

$$V_yC = V_y + \left.\frac{dV_y}{dt}\right|_{\substack{x = \frac{x_o+x_p}{2} \\ y = \frac{y_o+y_p}{2}}} \cdot \Delta t \qquad (76)$$

$$x_c = x_o + V_xC \cdot \Delta t$$

$$y_c = y_o + V_yC \cdot \Delta t$$

The equations (76) are iterated, replacing the x_p, and y_p values by the corrected values x_c and y_c until the successive values are within a preselected tolerance.

For the cases of very small Stokes number, it becomes necessary to adopt very small time steps to compute the trajectory accurately in any flow region with steep velocity changes (near the sampler head, for example). To overcome this difficulty, the velocities of particle Vx and Vy can be approximated and the following procedure is used.

$$Vx = Ux - K \frac{\partial Ux}{\partial x} \cdot Ux + VGx$$

$$Vy = Uy - K \frac{\partial Uy}{\partial y} \cdot Ux + VGy \qquad (77)$$

where

Vx = particle velocity in x direction

Vy = particle velocity in y direction

Ux = fluid velocity in x direction

Uy = fluid velocity in y direction

K = Stokes number of the particle

VGx = sedimentation velocity in the x direction

VGy = sedimentation velocity in the y direction

$$\frac{dx_p}{dt} = Vx$$

$$\frac{dy_p}{dt} = Vy \qquad (78)$$

where

x_p, y_p = coordinates of particles

t = time coordinate

From (78) we get

$$\frac{dy_p}{dx_p} = \frac{Vy}{Vx} \qquad (79)$$

Equations (77) and (78) are used to compute the trajectory of the particles.

Particle velocities Vx, Vy are computed from the equation (77). By using (79), the new position of the particle is predicted as:

$$Y_{PN} = Y_{PO} + \left.\frac{Vy}{Vx}\right|_{XPO} \cdot \Delta x \qquad (80)$$

where

YPN = new position of particle in Y

YPO = old position of particle in Y

Δx = a preselected step size in x

XPO = old position of particle in x

If the ratio of Y_{PN}/Y_{PO} is greater than 1.05, then the step size x is reduced until this criteria is met. Then a corrected new position of the particle is computed by calculating the velocities at the new position by equation (77) and using an average value of the velocity ratios in equation (80). That is

$$Y_{PN}(\text{corrected}) = Y_{PO} + \frac{1}{2}\left(\left.\frac{Vy}{Vx}\right|_{old} + \left.\frac{Vy}{Vx}\right|_{new}\right) \cdot \Delta X \qquad (81)$$

If we denote the corrected value of Y_{PN} as Yo then Equation (81) is iterated until Yo converges. That is

$$Yo^n = Yo^{n-1} + \frac{\left(\left.\frac{Vy}{Vx}\right|_{old} + \left.\frac{Vy}{Vx}\right|^{n-1}\right)}{2} \cdot \Delta X$$

where superscript n represents iteration number. The procedure is stopped when the successive values of YO are within a chosen error limit.

The above procedure is repeated until the particle passes the probe inlet, at which point it either gets captured by the probe or escapes it.

Calculation of the Efficiency of the Sampling

Let C_0 denote the number of particles/unit volume in the free stream and let C be the actual number of particles sensed by the instrument, then

$$\eta = \frac{C}{C_0} = \frac{C_I}{C_0} \cdot \frac{C}{C_I}$$

where

η = the efficiency of the sampling, and

C_I = the concentration at the inlet to the probe

$$\frac{C_I}{C_0} = E \cdot \frac{U_0}{U}$$

where

E = the efficiency of capture of inlet

$$\frac{C}{C_I} = \frac{C_I - \Delta C}{C_I}$$

where

ΔC = the loss of particles in the probe.

In the case of a polydispersed system, the distribution can be divided into n number of small groups. Each one can be treated in the same way as described above. The effective η will be given as

$$\eta_{eff} = \sum_{i=1}^{n} \eta_i f_i$$

where

η_i = the efficiency of the i^{th} group

fi = the fraction of i^{th} group, and

n = the number of groups.

In most of the cases, the measured size distribution under non-ideal conditions

are available and one likes to find the actual distribution. Denoting the fraction of i^{th} group by 'fi act', then:

$$fi_{act} = \frac{fi/\eta i}{\sum_{i=1}^{n} \frac{fi}{\eta i}} \qquad (82)$$

COMPUTER PROGRAM SYSTEM

The computer program system consists of two separate programs. Program 'FLOWFI' solves for the flow field and program 'TRAJEC' computes the particle trajectories in the specified flow region. Since the model uses the stream function equation (14) only when the sampling head faces the stream, program 'FLOWFI' has to be run only with this option. For angular orientations the flow field is approximated by a line sink/source in a uniform stream and the flow field is incorporated in the 'TRAJEC' program. A description of both programs follows.

Program 'FLOWFI'

This program to solve for the values of Stokes/Lagrange stream function ψ and to calculate the fluid velocity components Ur, Uz has been written in Fortran V for Univac 1108 digital computer. The program solves the flow fluid in and around the sampling head with circular/parallel plate geometry. The thickness of the sampling head W, chamfer angle α, and the velocity of suction ratio U are treated as parameters. An explanation of various program subroutines follows.

The calling sequence of the subroutines according to the user option of the flow field is accomplished in the Main Program.

Subroutine FLBOUN, meaning FLow BOUNdary, specifies the boundary of the flow field in the upstream (ZM), downstream (ZMA), and the ratial (RO) direction. It also specifies the sampling velocity ratio (U), chamfer angle (ANG) and the sampler wall thickness (W). The values of the upstream boundary ZM, downstream boundary ZMA, and the radial boundary RO have to be chosen arbitrarily. Typical nondimensional distances are ZM=5, ZMA=5 and RO=5 for sampling heads with 'W' $\leq$ 0.2. For very thick-walled tubes, these boundary values have to be increased so as to realize the uniform undisturbed flow condition.

The above values are input to the program and have to be supplied by the user. The subroutine calculates a value (ZOW) which is the axial coordinate of the outer edge of the sampler for use in further calculations.

Subroutine GRID places a grid of specified grid spacing in axial (Z) and radial (R) direction and calculates the coordinates of a given point. It also calculates the maximum number of points in axial direction (IM) and maximum number of points in radial direction (JM). The total number of grid points in the flow field would be IM x JM. The coordinates of any given point (I,J) would be given by (Z(I),R(J)). ITW is an indicator for the sampler wall

thickness. ITW:0 for W=0, ITW:1, for finite W and ANG-0 and ITW:-1 for finite W and finite ANG. Calculation regions for various options are shown in Figure 15. IGR, and JGR are number of grid points per unit distance in the axial direction and radial direction.

Subroutine BCOND formulates the boundary conditions of the problem. The boundary conditions are to be formulated at Section I, II, and III. The sampler wall conditions are also stipulated. Figure 16 gives the values of PSI at the boundary for circular tube.

Subroutine LAPLA solves the Stokes' stream function equation formulated for the problem earlier by using the boundary conditions provided by BCOND. The solution for the flow field is provided at the grid points specified by GRID. Successive over relaxation procedure is used to approach the solution. The iteration procedure is stopped either when the successive values of PSI are within the specified error tolerance (EPPS) or when the number of iterations has exceeded the maximum number of iterations (ITERMA) specified by the user. The relaxation factor (RELAX) has to be supplied by the user and the value varies from 1.0 to 2.0 depending upon the conditions of the problem. Optimum value has the be found by trial and error. The finite difference equation formulated in the previous sections is used. This equation is solved at each grid point of the flow field except the boundary points specified by BCOND. The maximum error occurring at each iteration is printed out under convergence rate.

Subroutine VELO calculates the axial and radial velocity components Uz, Ur, respectively, from the stream function solution provided by the subroutine LAPLA. Velocities are calculated by determining by second order accurate finite difference expressions. Depending upon the position of the grid point, either one of the following finite difference forms is chosen:

1) ESC (Equal Spaced, Centered)
2) ESB (Equal Spaced, Backward)
3) ESF (Equal Spaced, Forward)
4) UESB (Unequal Spaced, Backward)
5) UESF (Unequal Spaced, Forward)
6) UESC (Unequal Spaced, Centered)

Subroutine RESULT prints out the results obtained from various subroutines. User can manipulate the output statements to suit his needs.

Subroutine STREAM calculates the locus of any specified stream line for use in the plotting of stream lines. A lagrange interpolation is used to calculate the locus. Function statement SAG is the lagrange interpolation formula for three points with unequal intervals.

The program listing is provided in Appendix D.

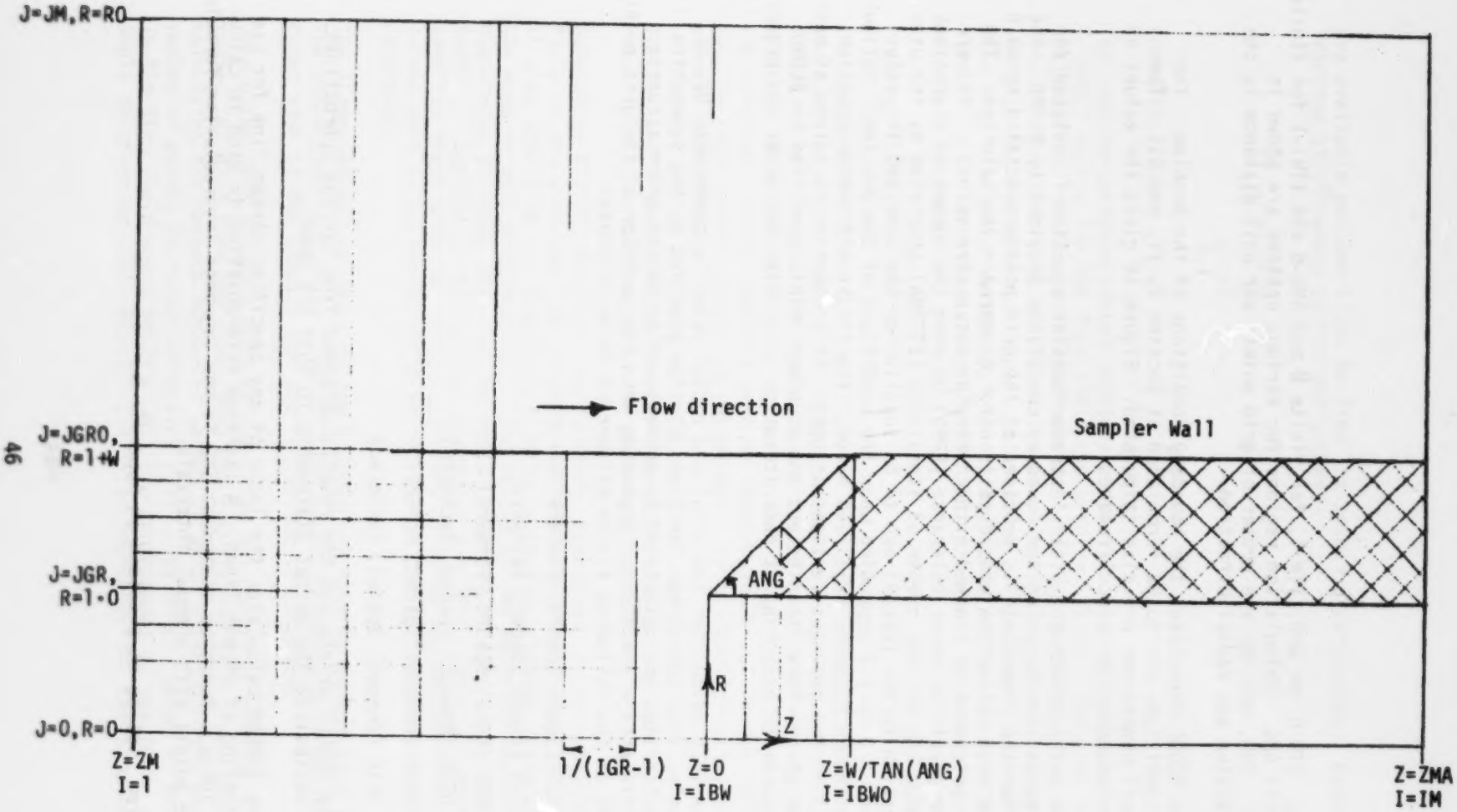
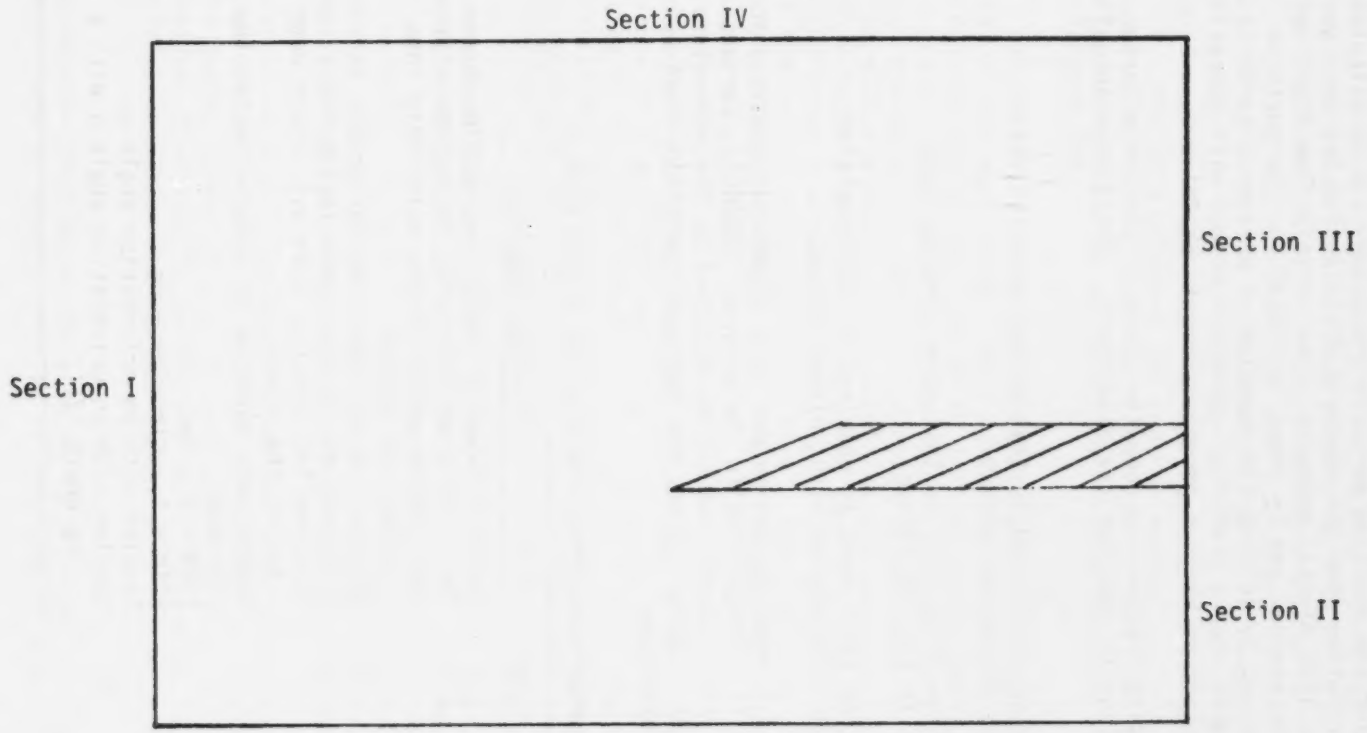


Figure 15. Flow field boundary and grid point layout.



Section	PSI
I	R(J)**2
II	U*R(J)**2
III	A*(R(J)**2-(1+W)**2)+U where $A = \frac{(RO^{**}2-U)}{(RC^{**}2-(1+W)^{**}2)}$
IV	RO**2

Figure 16. Boundary conditions for circular tube.

Program 'TRAJEC'

This program to solve for the limiting particle trajectory and to calculate the true particle distribution from the measured distribution has been written in Fortran V for Univac 1108 digital computer. For certain flow field options, the flow has to be predetermined and is input to 'TRAJEC'. The particle trajectories are determined by solving the equation of motion of particle with Stokes' drag. The program uses a predictor-corrector method with iterative convergence. A description of various program subroutines follows.

Program 'INER' is the Main Program and computes the particle trajectories. The user has three options in choosing the flow field. These are controlled by an integer 'NDIM'.

NDIM = 0 The flow field used is that of two parallel plate inlets facing the stream.

NDIM = 1 The flow field used is that of a circular tube inlet facing the stream.

NDIM = 2 The flow field used is obtained by superposition of line sink/source with uniform stream.

Choosing the option 0 or 1 requires the output from 'FLOWFI'. However, for option 2 the flow field is incorporated in the program 'TRAJEC'. Various inlet geometries and physical conditions can be obtained by the parameters U, W, ANG, NDIM, IDIM, ALPHA. Table 3 shows the physical conditions that can be obtained by various combinations.

Table 3. Parameter desription and physical significance.

NDIM	W	ANG	ALPHA	ZLIP	Physical Meaning
0	0	0	90°	0	Two D thin-walled plates facing the stream.
	Wo	90°	90°	0	Two D thick-walled plates facing the stream.
	Wo	α	90°	0	Two D thick-walled plates with sharp edge facing the stream.
1	0	0	90°	0	Thin-walled circular tube facing the stream.
	Wo	90°	90°	0	Thick-walled circular tube facing the stream.
	Wo	α	90°	0	Thick-walled circular tube with sharp edge facing the stream.
2	-	-	90°	-	Approximate solution to sampler facing the stream. IDIM = 0 - Two • D IDIM = 1 - circular tube
	-	-	α	0	Sampler with an orientation angle α.
	-	-	α	ZL	Sampler with an orientation angle α with a lip depth 'ZL'.

Program TRAJEC starts the trajectory calculation at position ZI, RI input by the user. ZI is the upstream distance at which the particle is moving with the stream and is usually at least 5 radii from the sampling head. RI is the radial position to start the process to find RC, the radial position of limiting trajectory at ZI. If the trajectory of the particle starting at position RI enters the probe then the next trajectory is started from a position RI = RI + DR, where DR is a preselected radial increment. If the trajectory from RI escapes the sampler then the new RI is given by RI-DR. This process continues until the trajectories from successive radial positions alternate (i.e., one gets captured and another escapes). This process is repeated with successive halving of DR until the radial positions RES (at which the particle escapes) and RCA (at which the particle gets captured) are within a preselected tolerance EPP.

To calculate the trajectory accurately it is necessary that the time or space step is not very large. This is accomplished by taking the ratio RAT of predicted radial position RP to initial radial position RO. If the ratio RAT is greater than a prespecified value CAT then the time/space step is halved and recalculation starts. The value of CAT used in the program is 1.05. Even with a step size that satisfies the ratio test if the iterative convergence fails within LIMIT iteration then the step size is halved and the calculation procedure is restarted. The origin of the coordinate system lies at the center of the sampling head. The Z axis is along the centerline and R axis along the face plane. The efficiency calculation is performed for IST number of particles.

Subroutine FCT calculates the derivatives of the velocities at a given position R, Z. When NDIM option of 2 is used, the fluid velocity can be calculated at any given point but for option 0 and 1 the velocities are available only at the grid points and interpolation is needed. Subroutine INTERP performs the interpolation and computes the fluid velocity at a given particle location. A two-dimensional linear interpolation is used.

Subroutine OUTP prints the particle position at various time intervals. The printing interval INT is chosen by the user. For example, when INT = 50, the result is printed once in every 50 steps. Subroutine OUTP also tests for the deposition, capture or escape, of particle from the sampler and is indicated by KODE. When KODE = 0, particle is escaped and when 1 particle is captured, KODE = 2 indicates that particle has deposited on the sampler wall.

Subroutine BIAS calculates the actual size distribution from the measured size distribution . It uses the efficiency calculated by the program INER. The Stokes number of a given particle K is also calculated by BIAS. The measured fraction PF, particle size P, relaxation time TAU are input to the subroutine BIAS. The maximum number of particle intervals that can be used is 10.

A computer listing of the program is provided in Appendix D. The users manual and an example problem is provided in Appendix E.

LIMITATIONS OF THE MODEL

(1) The model uses inviscid flow pattern around the sampler to compute particle trajectories. Close to the sampler wall the effect of boundary layer dominates the flow pattern and modifies the particle trajectories.

(2) The error in collection efficiency for particles with much smaller Stokes number K (<0.001) is high when using the flow field option NDIM=2. The main cause of the error is the deviation of the model flow lines to the actual flow lines.

(3) The effect of the physical presence of sampler and inlet geometry are not incorporated in the model for angular orientations of the sampler (option NDIM=2).

RESULTS AND DISCUSSION

Even though there are a number of theoretical and emperical models available to estimate the sampling errors due to anisokinetic sampling they fail to account for the effect of sampler orientation. The major difficulty in incorporating the effect of orientation is solving the flow field. This is circumvented by approximating the flow field around the sampling head with a two-dimensional line sink superposed on uniform flow. Such an assumption will render the exact flow field when the sampler inlet is a slit but for other inlet geometries the assumption is valid in the core region of the inlets. Even with such a simple model, one can gain a physical insight to the effect of sample orientation.

The computer program system was used to obtain the sampling bias for various particles when sampled by a thin-walled circular tube. Sampler inlet geometry of thin-walled circular tube approximates the flow around the vicinity of the closed face filter used in personal sampling. The velocity ratios used are: 1.2; 2.0; 0.75; 0.375; 0.1875; and 0.0938.

The concentration ratios were calculated at least at 8 Stokes numbers K. The results are given in Figures 17-22. The X axis represents the ratio of measured concentration C/actual concentration C_0. The Y axis represents particle Stokes number K. For all the cases sedimentation and electrostatic effect has been neglected.

Particles with negligible inertia ($K \to 0$) move with the fluid stream, thus the concentration ratio C/C_0 approaches unity. But particles with infinite inertia ($K \to \infty$) continue in their original direction of motion and the concentration ratio C/C_0 approaches the velocity ratio U_0/U.

At velocity ratios close to one such as 1.2 or 0.75, the maximum error in concentration measurement is approximately 30%. But at extreme velocity ratios the error becomes as high as 100% for super isokinetic velocity ratios (>1) and infinite at subisokinetic velocity ratios (<1). Hence, sampling error for subisokinetic velocity ratios are more important and significant than the error for superisokinetic velocity ratios.

For the case of open-faced filter sampling, flow field option of NDIM=2 was used. In calculating the limiting trajectory of particles, the filter cassette was considered to have a lip depth of 0.2 x radius. The sampling bias for various particles when sampled at an angle α to the oncoming stream was calculated for a velocity ratio of 0.375.

Figures 23 through 27 show the trajectory of particles when sampled at a velocity ratio of 0.375. Effect of Stokes number of the particle trajectory is shown in Figures 23 through 25 at the orientation angle of 90°. Flat trajectories are characteristic of large Stokes number K. Figures 26 and 27

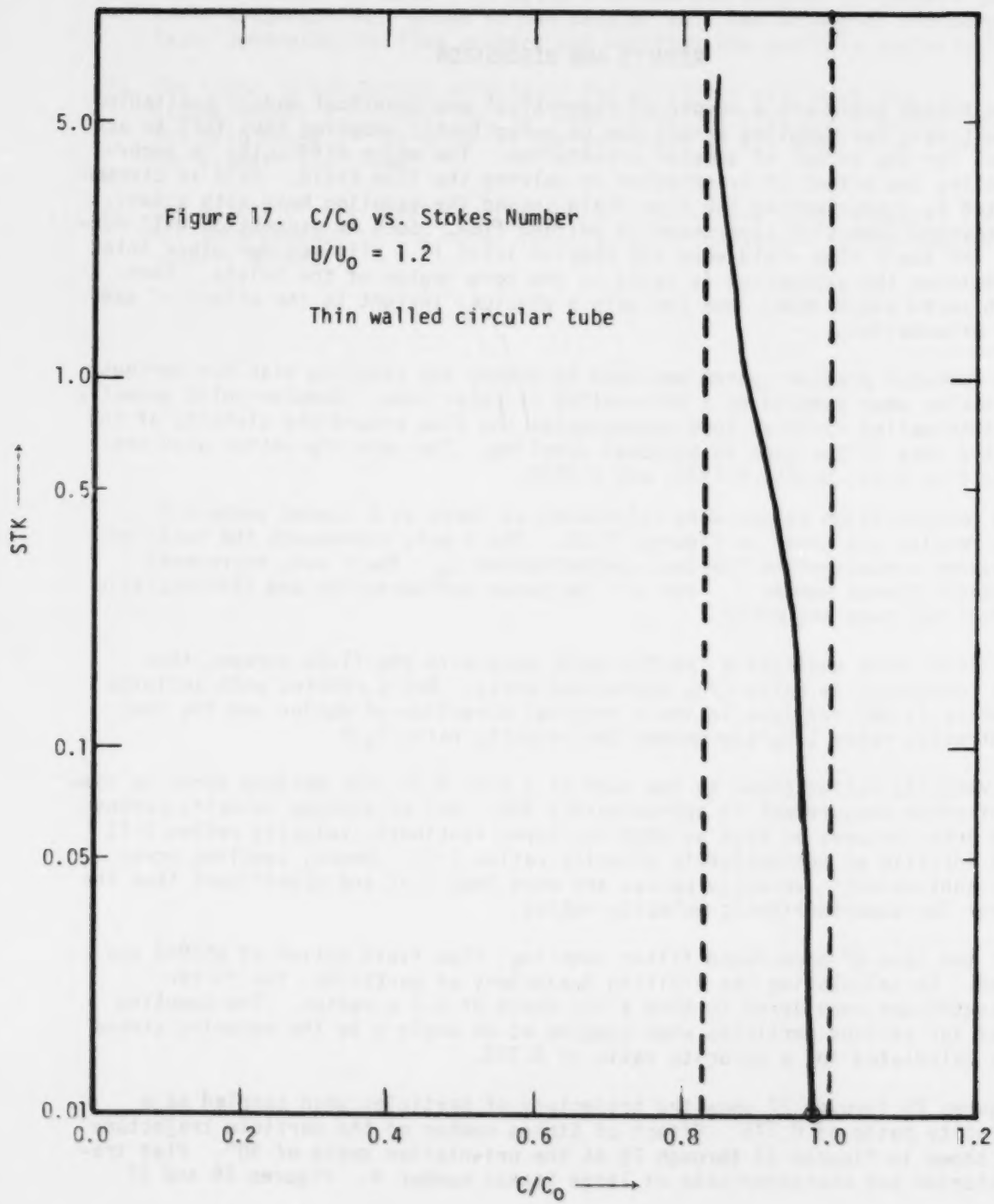


Figure 17. C/C_o vs. Stokes Number
$U/U_o = 1.2$

Thin walled circular tube

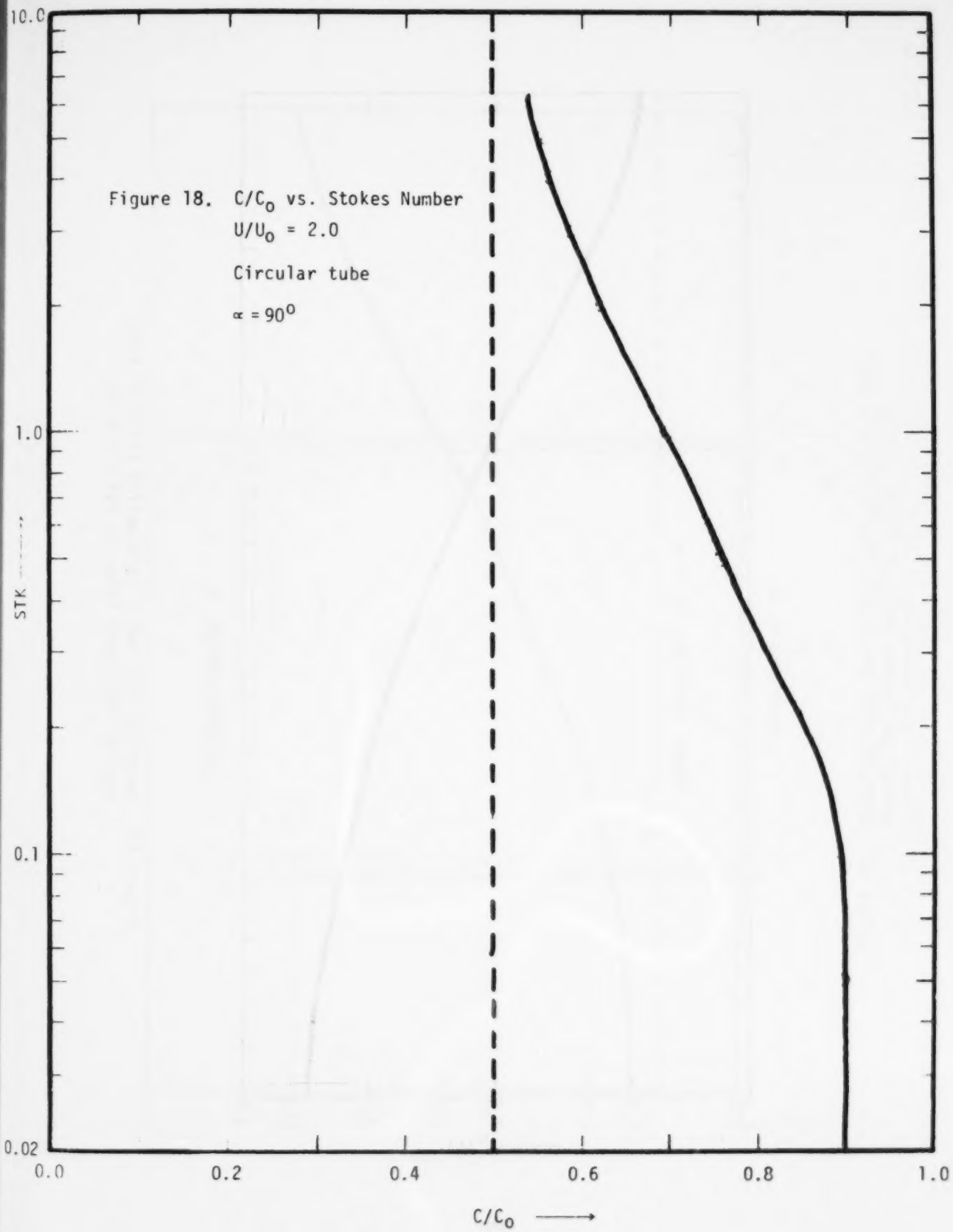


Figure 18. C/C_o vs. Stokes Number
U/U_o = 2.0
Circular tube
$\alpha = 90^o$

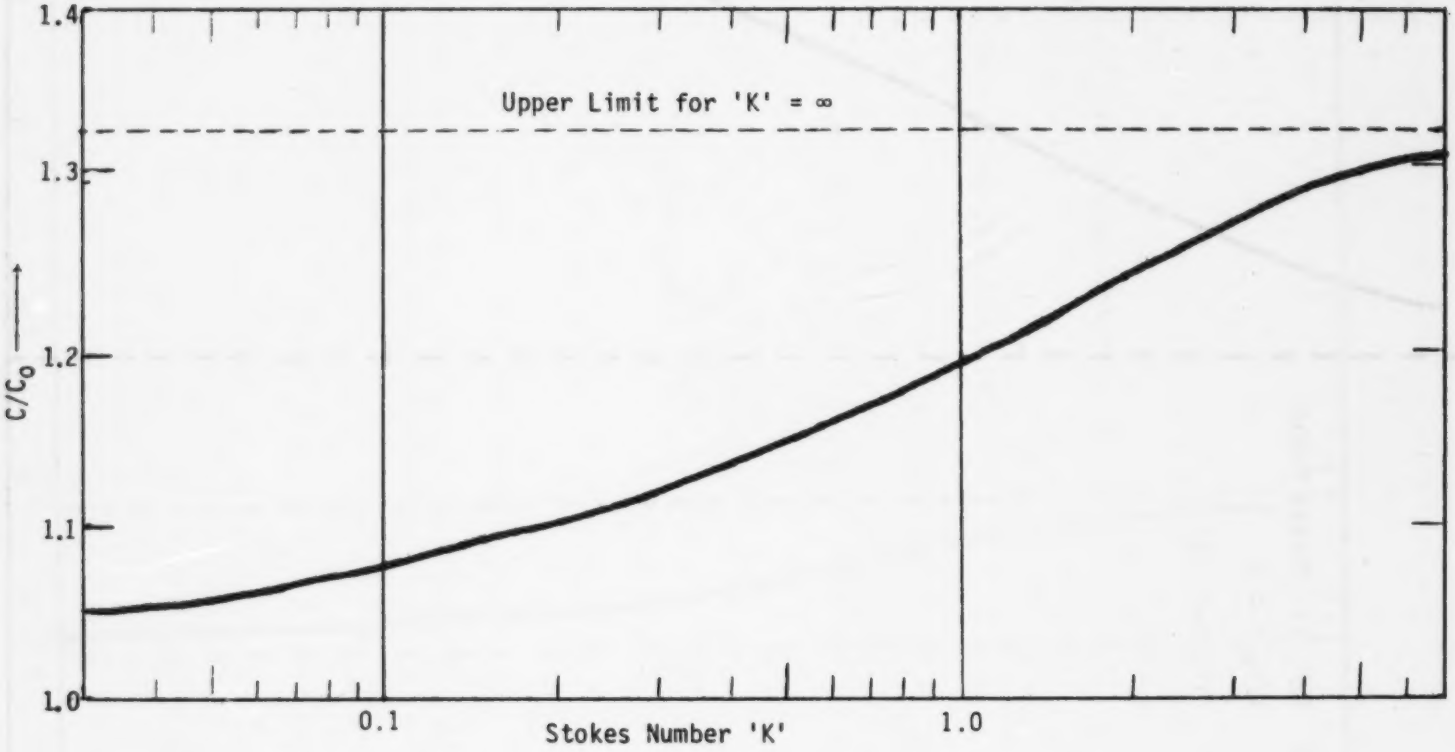


Figure 19. Sampling bias for a thin-walled circular tube facing the stream velocity ratio = 0.75.

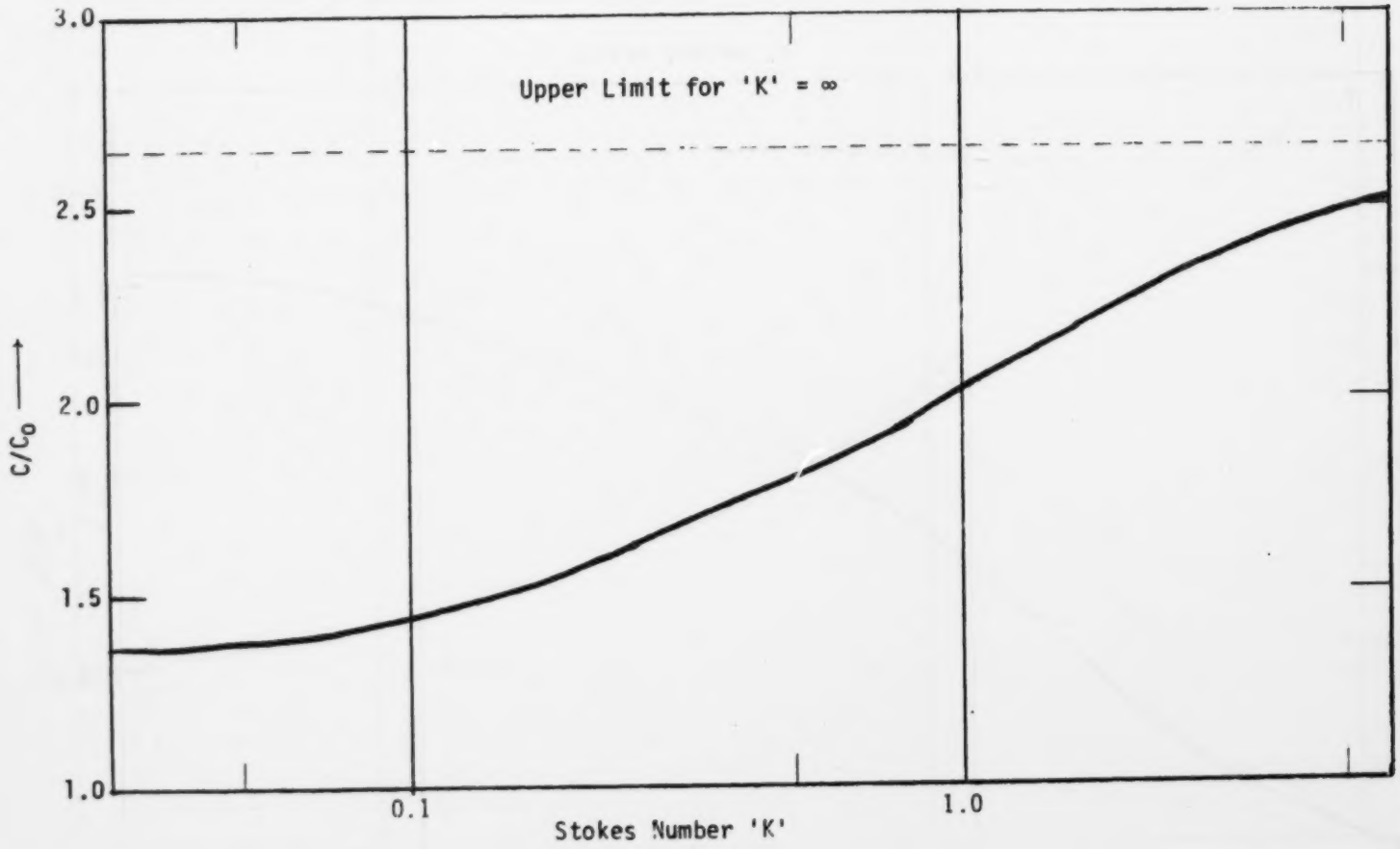


Figure 20. Sampling bias for a thin-walled circular tube facing the stream velocity ratio = 0.375.

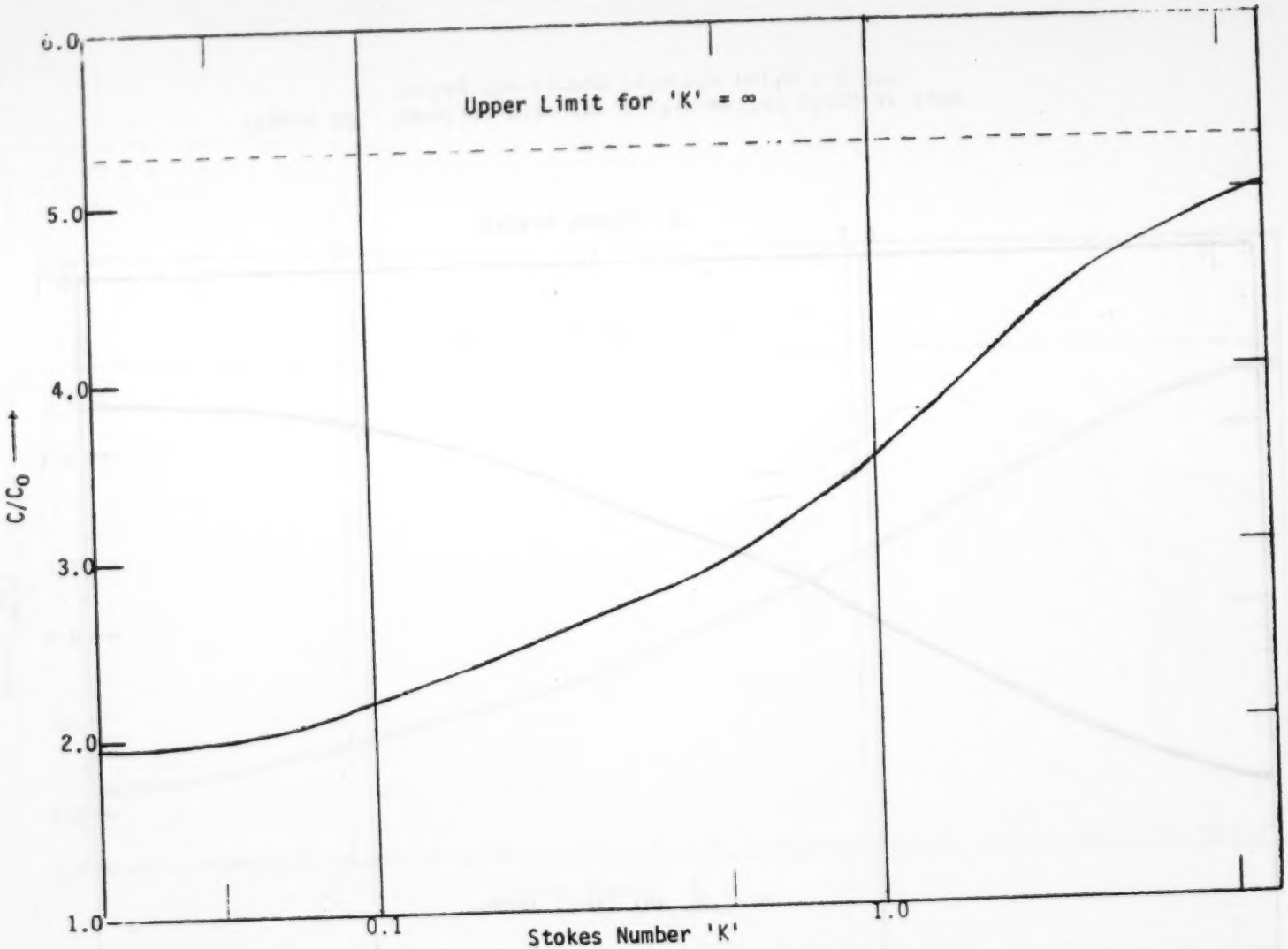


Figure 21. Sampling bias for a thin-walled circular tube facing the stream velocity ratio = 0.1875.

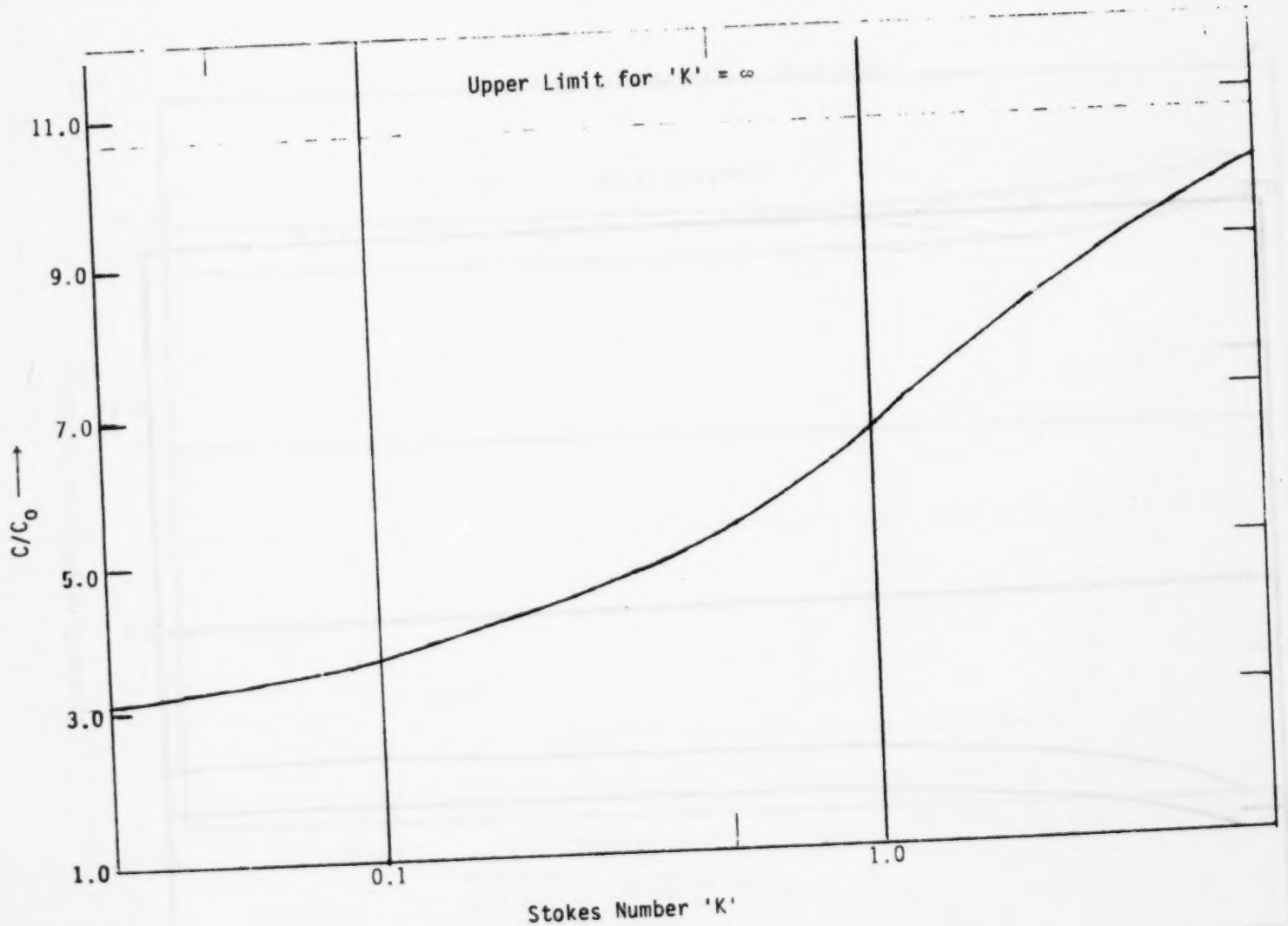


Figure 22. Sampling bias for thin-walled circular tube facing the stream velocity ratio = 0.0938.

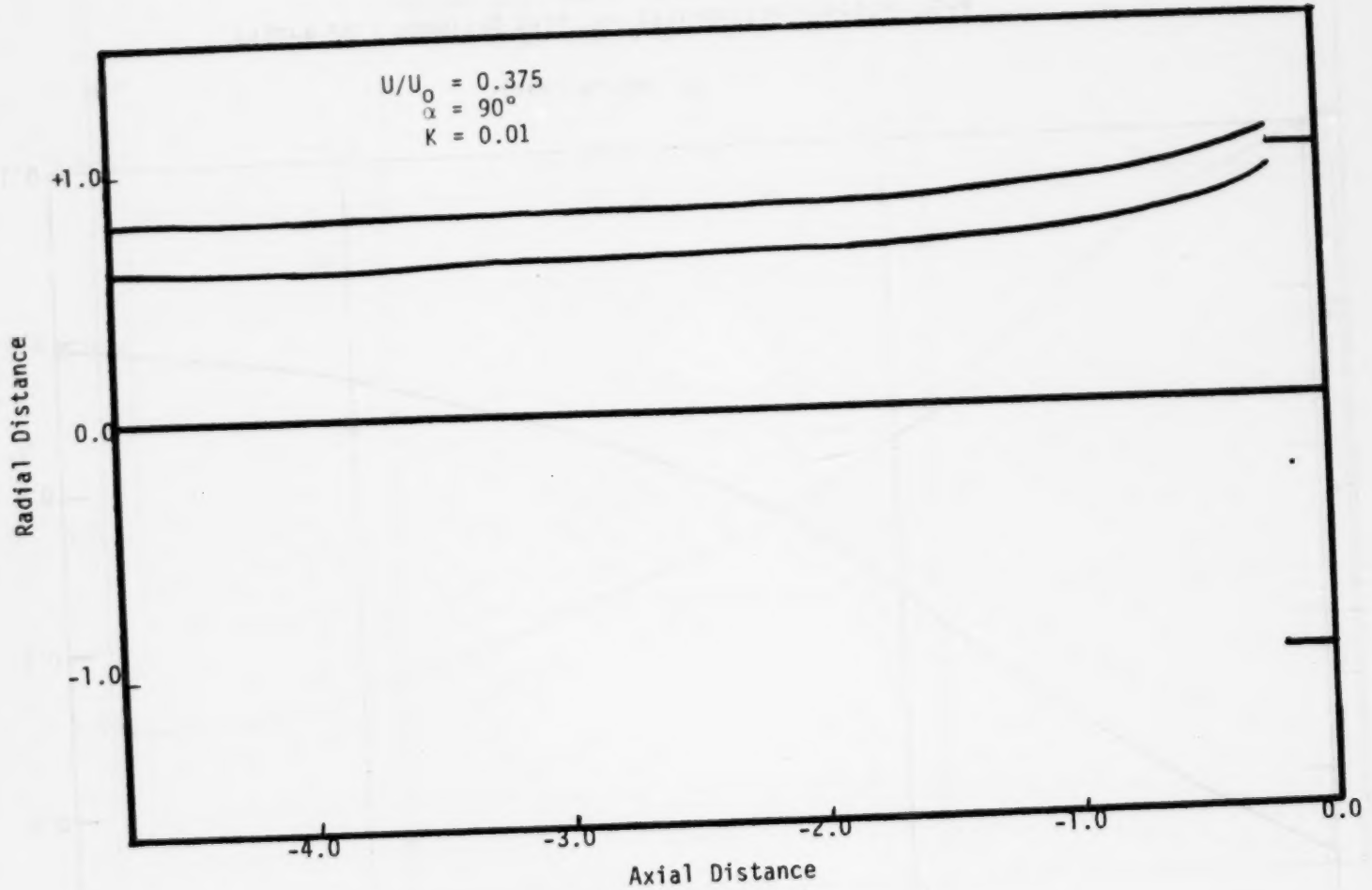


Figure 23. Trajectory of particles.

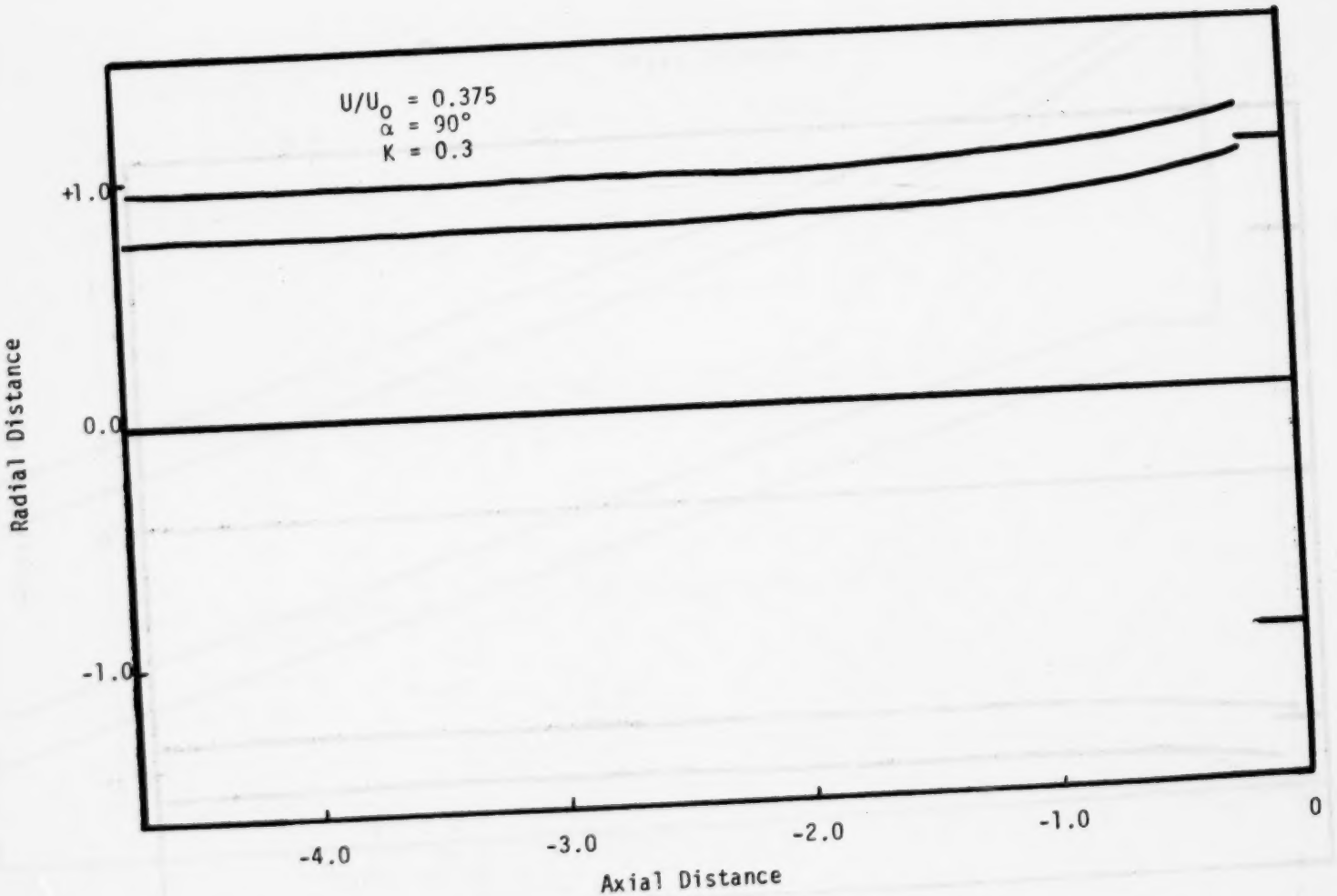


Figure 24. Trajectory of particles.

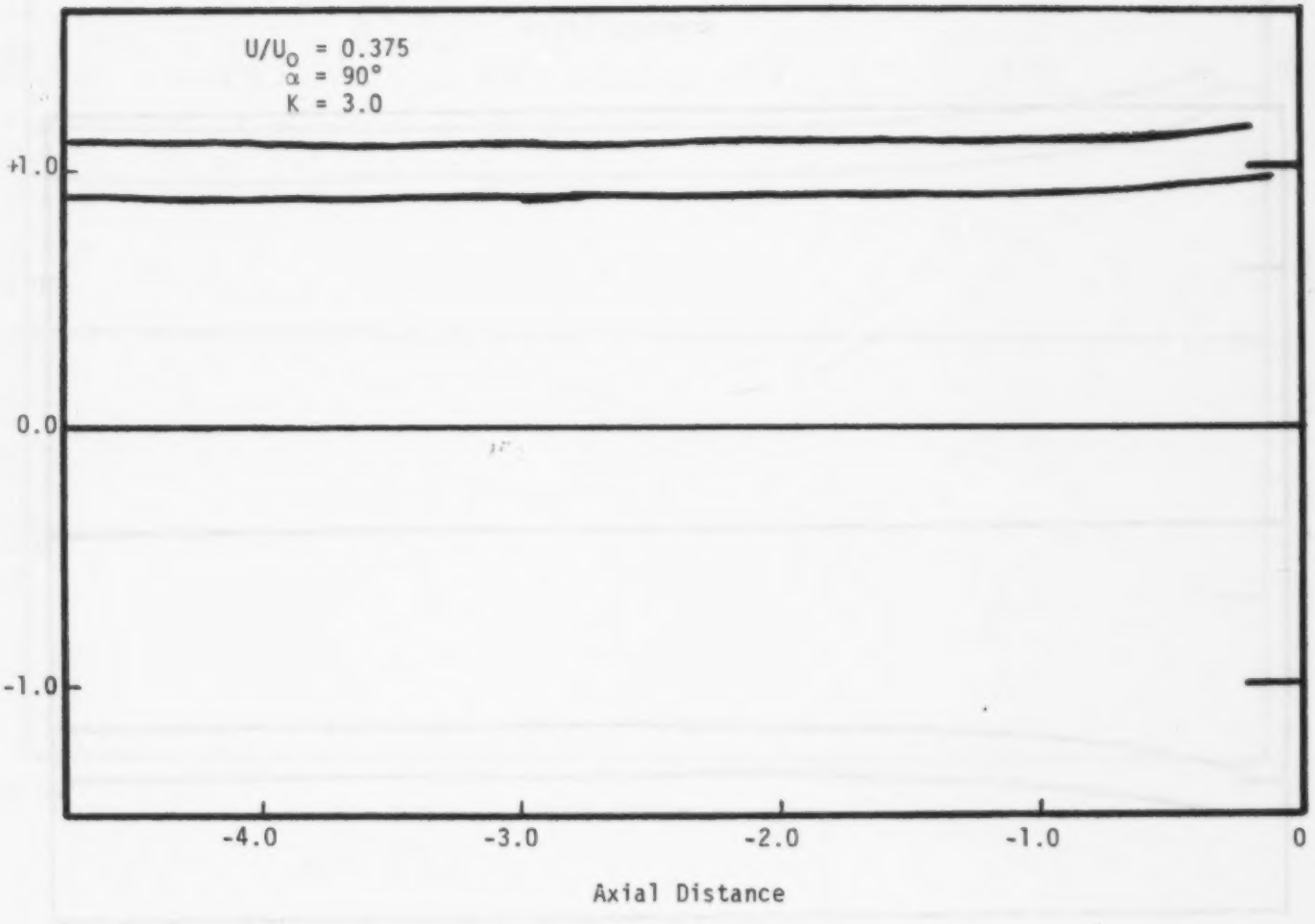


Figure 25. Trajectory of particles.

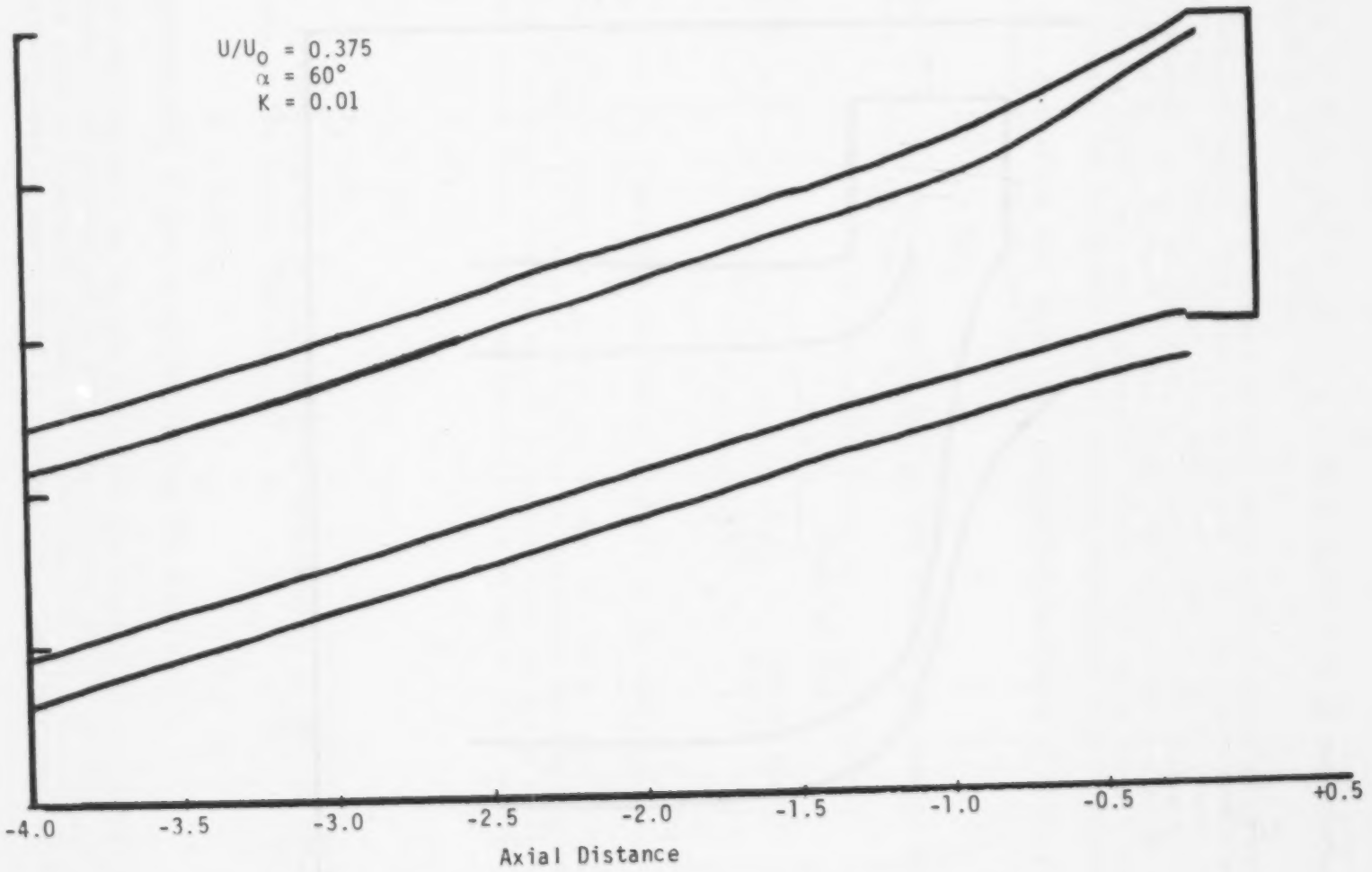


Figure 26. Trajectory of particles.

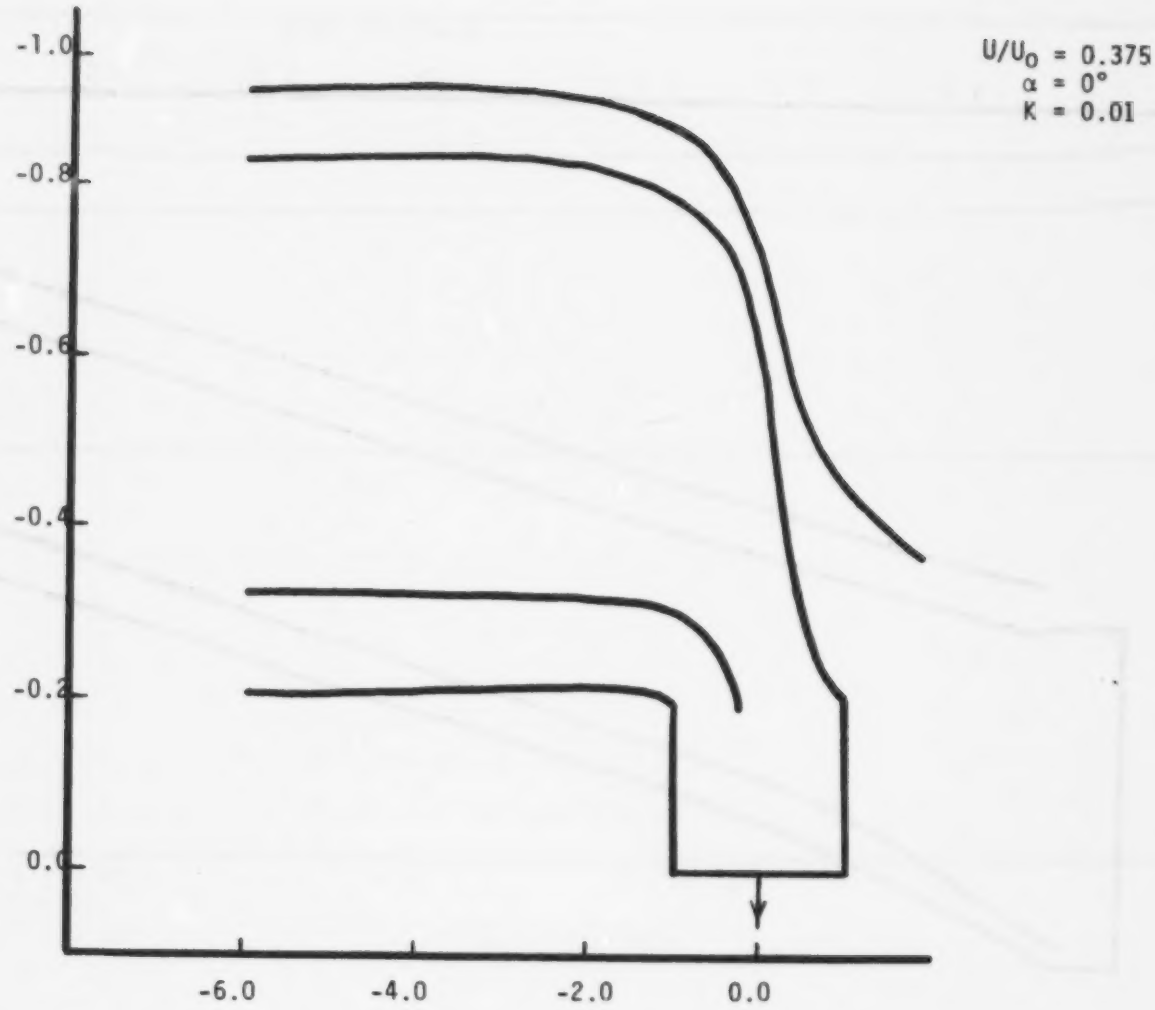


Figure 27. Trajectory of particles

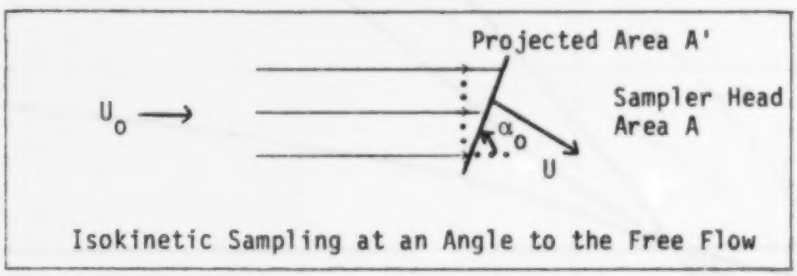
show the trajectory of a particle of Stokes number 0.01 at the orientation angles of 60° and 0°, respectively.

Figure 28 shows the efficiency of sampling of various particles at different orientation angles. The velocity ratio is 0.375. The theoretical limits of efficiency for Stokes numbers of ∞ and 0 are shown in solid lines. The experimental values of efficiency for a particle of Stokes number 0.553, as obtained by G.S. Raynor[30] is also shown. The values predicted by the current model are higher than the experimental values. The reason for over estimation is probably due to effects such as particle bounce-off, flow turbulence, etc.

Figure 28 predicts that there is an angular orientation α_o at which all the particles are sampled isokinetically.

$$\text{Sin } \alpha_o = U/U_o \tag{83}$$

This holds only when the sampler wall is very thin and the sampler body doesn't affect the flow.



Isokinetic Sampling at an Angle to the Free Flow

The physical meaning of equation (83) is that the correct tilt of the sampler α_o can reduce the volume of air sampled to compensate for sampling velocities $U<U_o$. Isokinetic sampling happens in general when the volume of fluid incident on the sampler cross-section equals the volume of fluid sampled:

$$V_i = V_s$$

where $$V_i = U_o A' \tag{a}$$

$$V_s = UA$$

A' is the projection of A on the plane perpendicular to the free flow, as shown in the figure. Therefore,

$$A' = A \sin \alpha_o. \tag{b}$$

Equation (83) is derived by substituting (a) into (b).

The sampling bias for various particles, when sampled anisokinetically by a square inlet was obtained by use of the computer program system. The results are shown in Figures 29 through 33. The flow field in the vicinity of the sampling head is approximated by superimposing a two-dimensional uniform flow

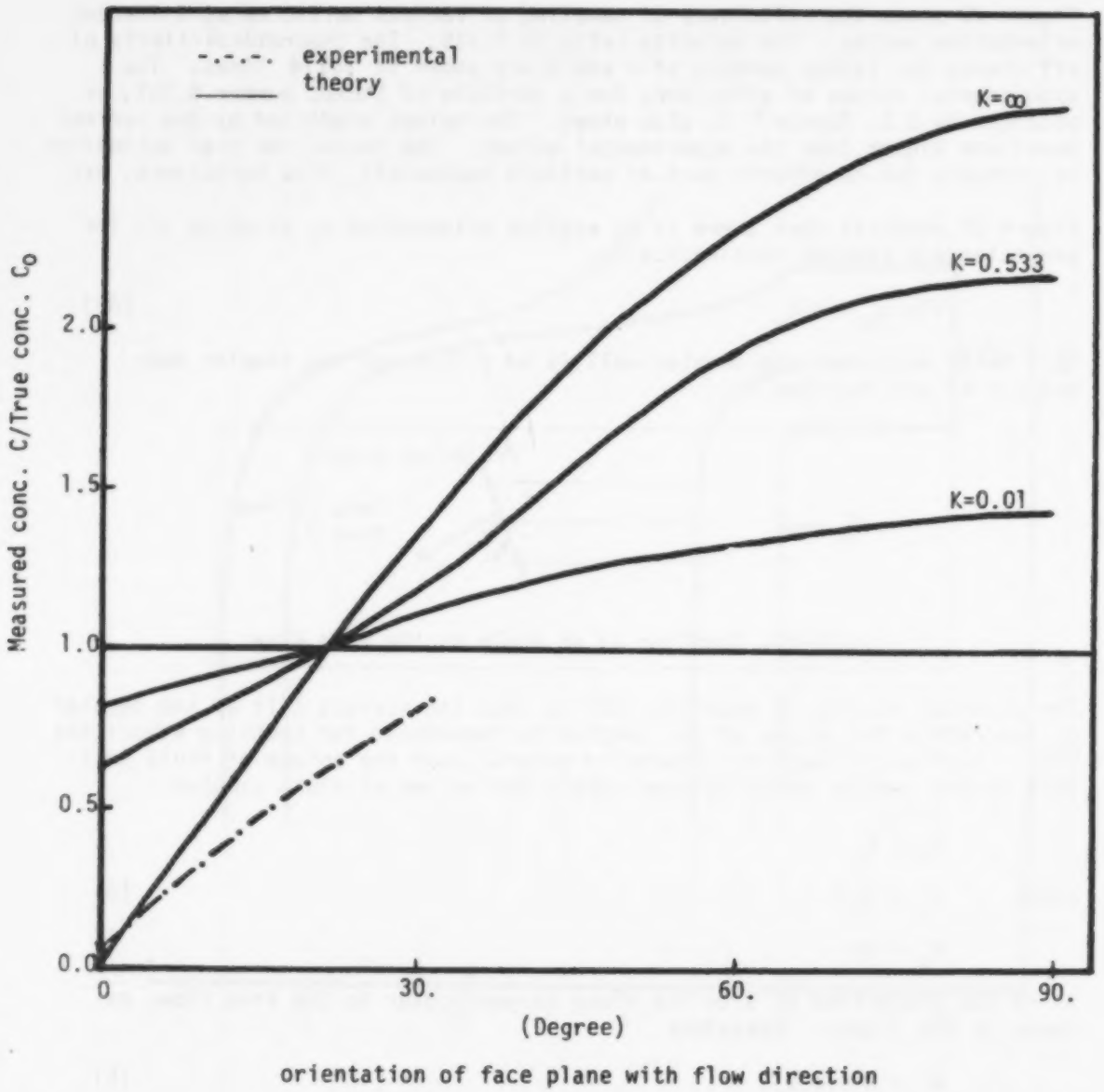


Figure 28. C/C_o vs. orientation angle for U/U_o = 0.375 and lip depth is 0.2.

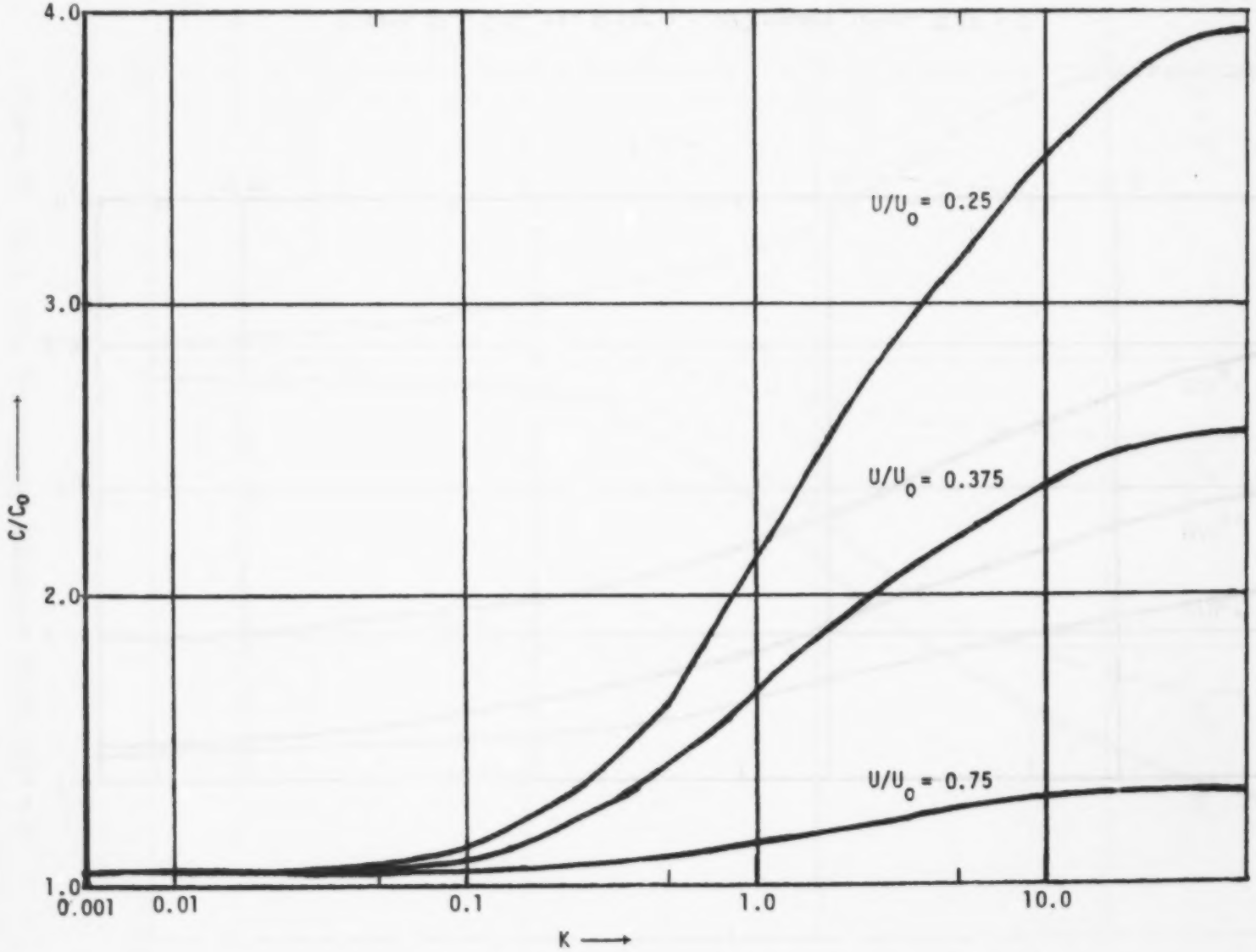


Figure 29. C/C_0 vs. K for $\alpha = 90°$ square inlet, ZLIP = 0.

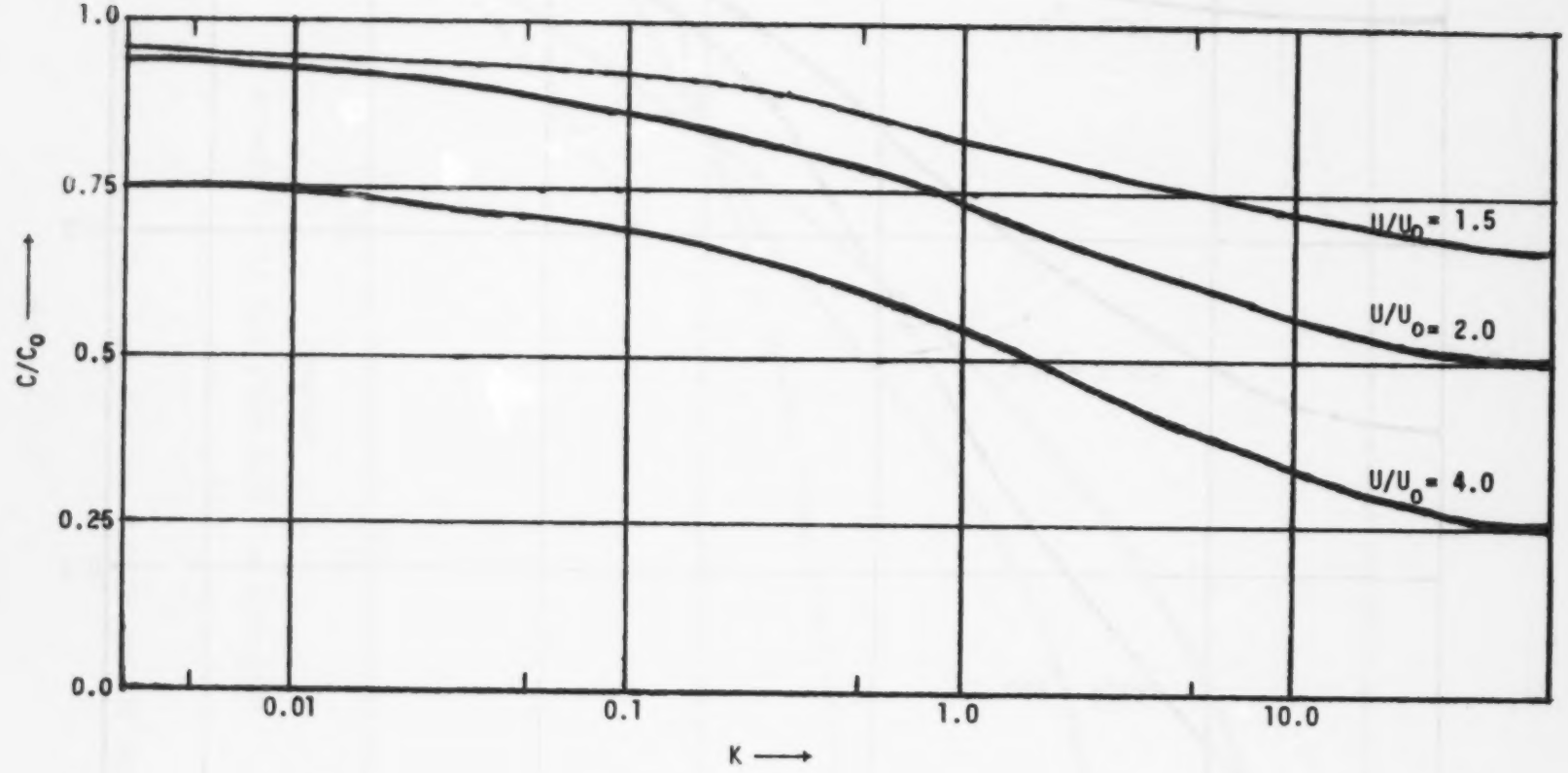


Figure 30. C/C_o vs. K for α = 90° square inlet, ZLIP = 0.

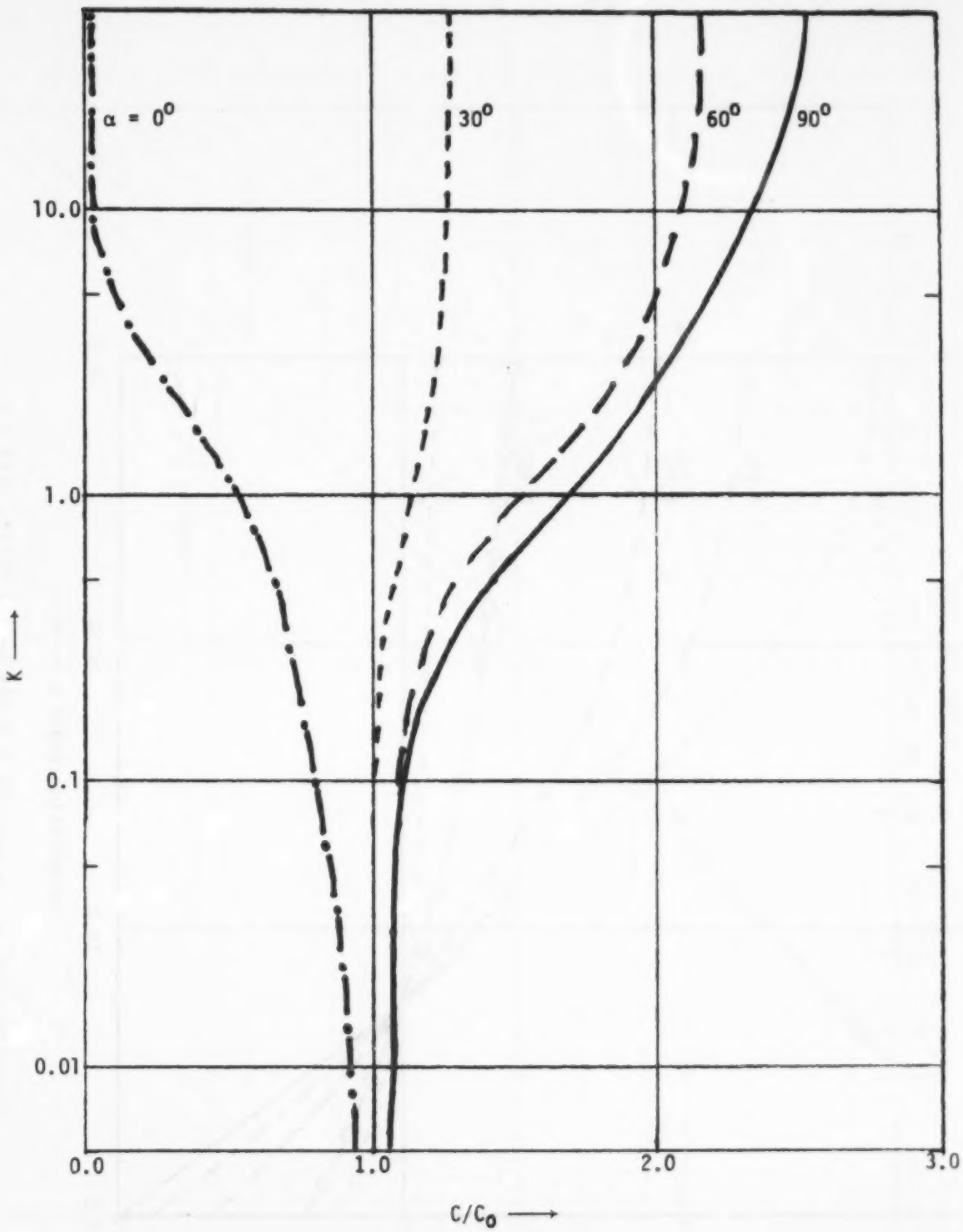


Figure 31. K vs. C/C_0 for U/U_0 = 0.375 square inlet, ZLIP = 0.

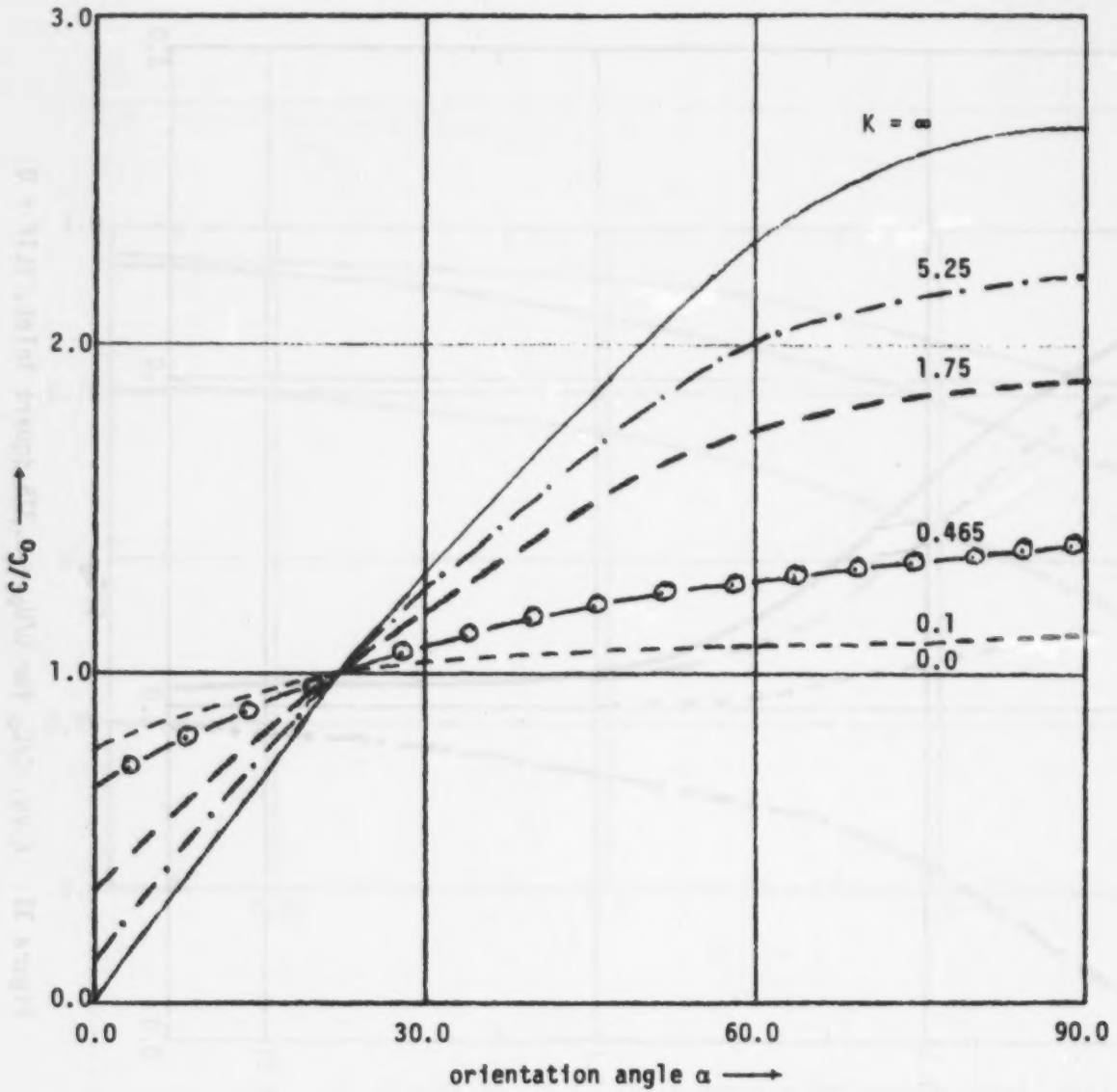


Figure 32. C/C_0 vs. α for U/U_0 = 0.375 square inlet, ZLIP = 0.

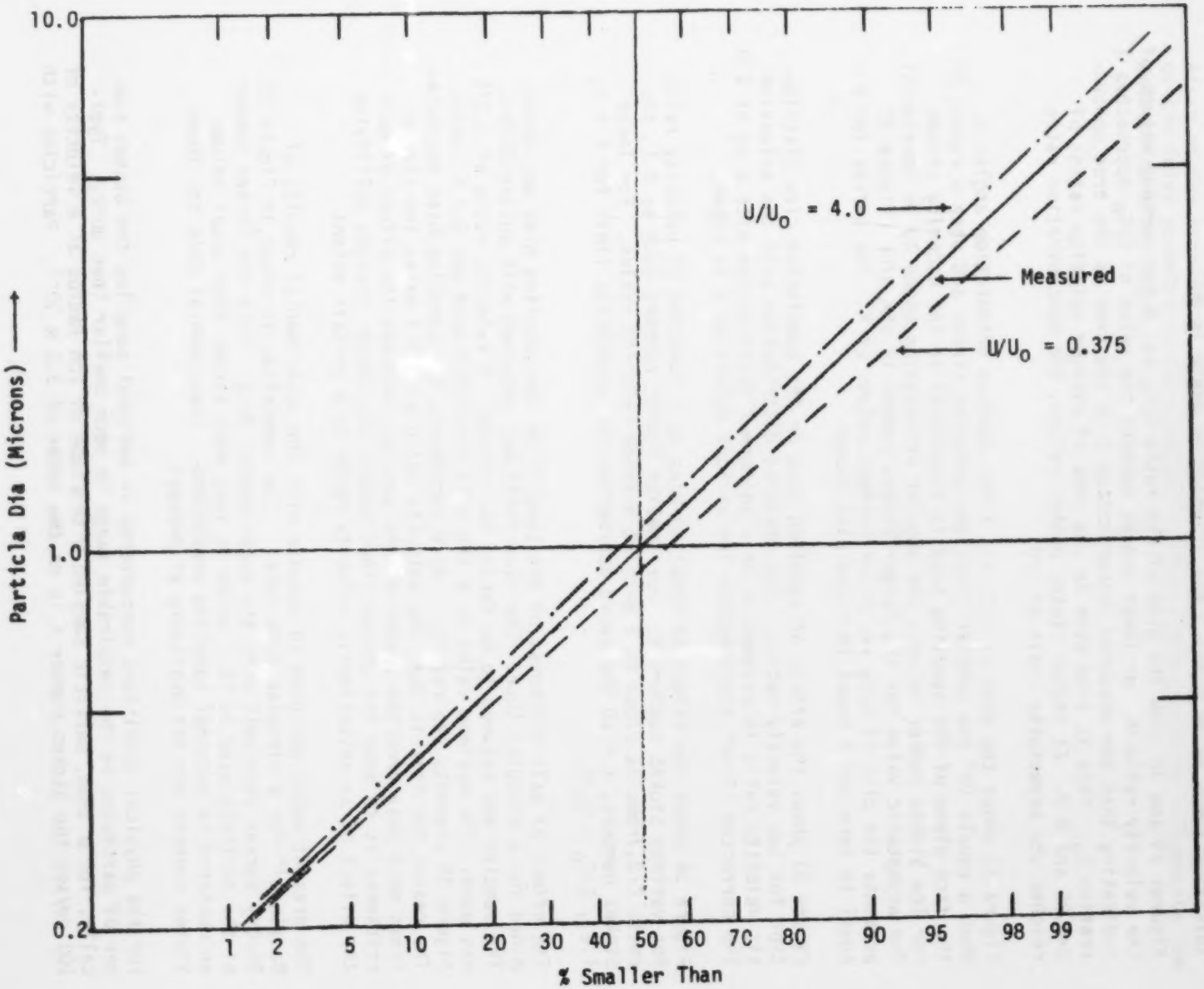


Figure 33. Effect of Bias on the Distribution $\alpha = 90^{o}$, square inlet, ZLIP = 0.

with a line sink located at the face of the inlet (NDIM=2). The lip length ZLIP is assumed to be 0 and α the orientation angle is 90°.

Figures 29 and 30 show the plot of the ratio C/C_0 vs. K for various values of the velocity ratio U. At lower Stokes numbers the value of C/C_0 approaches 1 indicating that the measured concentration C is the same as the true concentration C_0. This is true even in the case of extreme velocity ratios of U=0.25 and 4.0. At higher Stokes number values, the concentration ratio reaches the asymptotic limit of 1/U.

Figure 31 shows the plot of C/C_0 vs. K for various orientation angles α. When α equals 90° the sampler faces the oncoming stream and when α equals 0° the face plane of the sampling head is tangential to the oncoming stream. For low Stokes number values, the angular orientation seems to be immaterial. The asymptotic value for the larger Stokes number is sin α/U. Figure 32 presents the plot of C/C_0 vs. α for various values of K. The curves for K equal to zero and K equal to ∞ are also known.

Figure 33 shows the effect of sampling bias on the cumulative size distribution for two velocity ratios. The measured distribution with the anisokinetic velocity ratio is assumed to be a lognormal distribution with a σg of 2.0. The corrected "True" distribution for various values of U is shown.

Figure 34 shows the effect of sampling bias as a function of velocity ratio for various Stokes numbers K. For smaller Stokes numbers such as 0.1, the ratio C/C_0 remains close to 1 even at extreme velocity ratios. For large Stokes numbers, K = 10 the curve approaches the asymptotic limit for $K = \infty$, i.e., $C/C_0 = U_0/U$.

The effect of wall thickness of the sampler on the sampling bias was determined for a circular tube. The flow field was obtained with option NDIM=1. The sampler was assumed to be facing the stream. A velocity ratio of 0.375 was used. The maximum value of W the wall thickness used was 0.2 x radius. Figure 35 presents the results. As W increases, the sampling bias decreases. The reason for this is that the velocity ratio of 0.375 makes the stream lines move away from the center of the sampler, whereas the effect of wall thickness is to move the stream lines toward the center thereby nullifying the effect of an anisokinetic velocity ratio to a certain extent.

The present model was used to compare with the experimental results of Badzioch[31] for a circular tube inlet. The comparison is shown in Figure 36. Theory agrees very well with the experimental data. Here the Stokes number of the particle used is 10.5 which is very much larger than usual values encountered in personal sampling conditions. Experimental data for lower Stokes numbers are not available at present.

For the physical conditions encountered in personal sampling the Stokes number of particles in the respirable range is much smaller than unity. Typically, for a 10μm, particle sampled with a tube of 1cm radius at a velocity of 100 cm/sec the Stokes number K is on the order of 3.1×10^{-2}. Particles with

such a small Stokes number will follow the fluid stream lines even if the curvature of the stream line is quite high. So the anisokinetic velocity ratio or the inlet geometry is not expected to play a major role in influencing the collection efficiency.

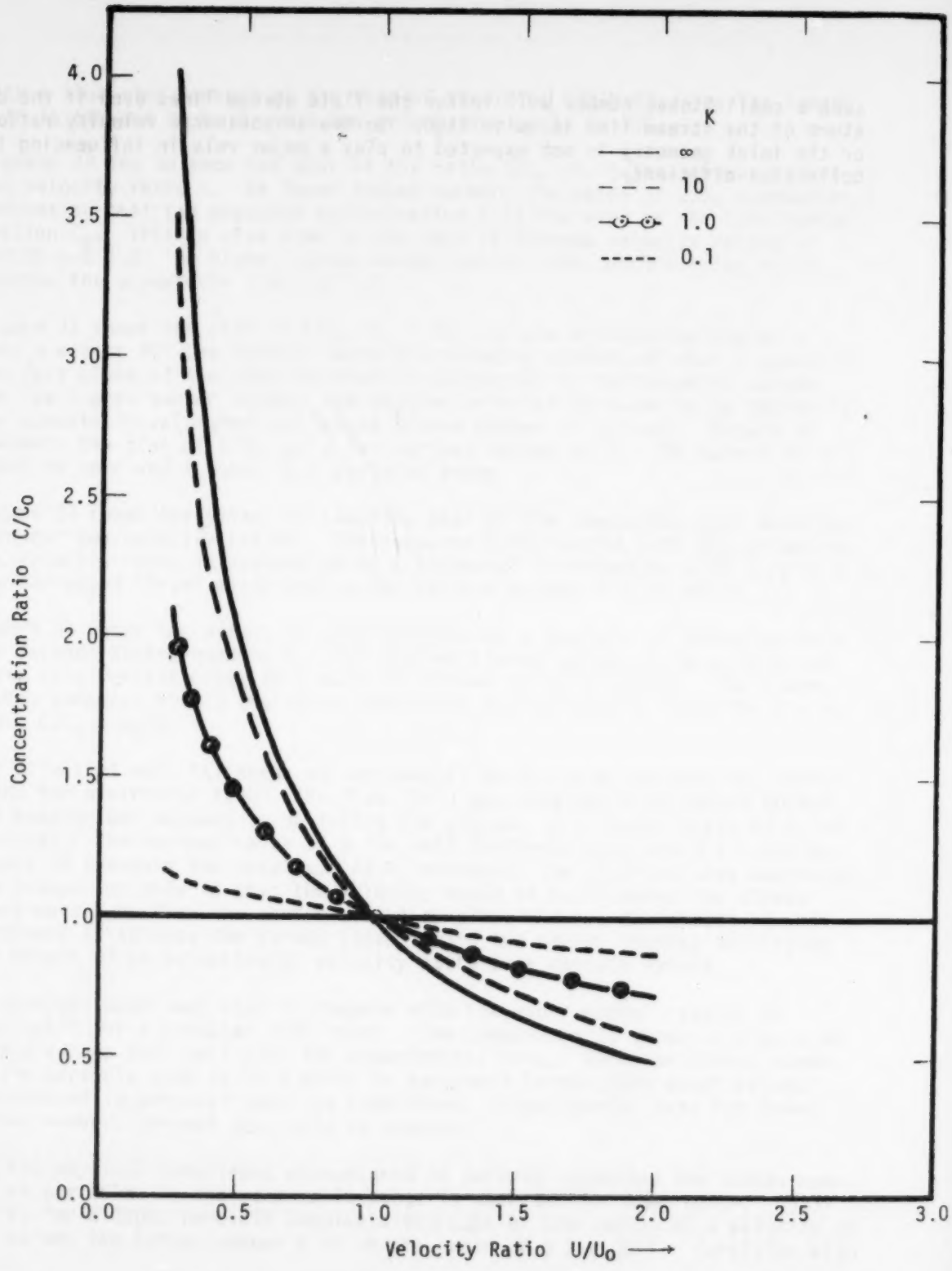


Figure 34. C/C_0 vs. U/U_0 for Square Inlet Facing the Stream.

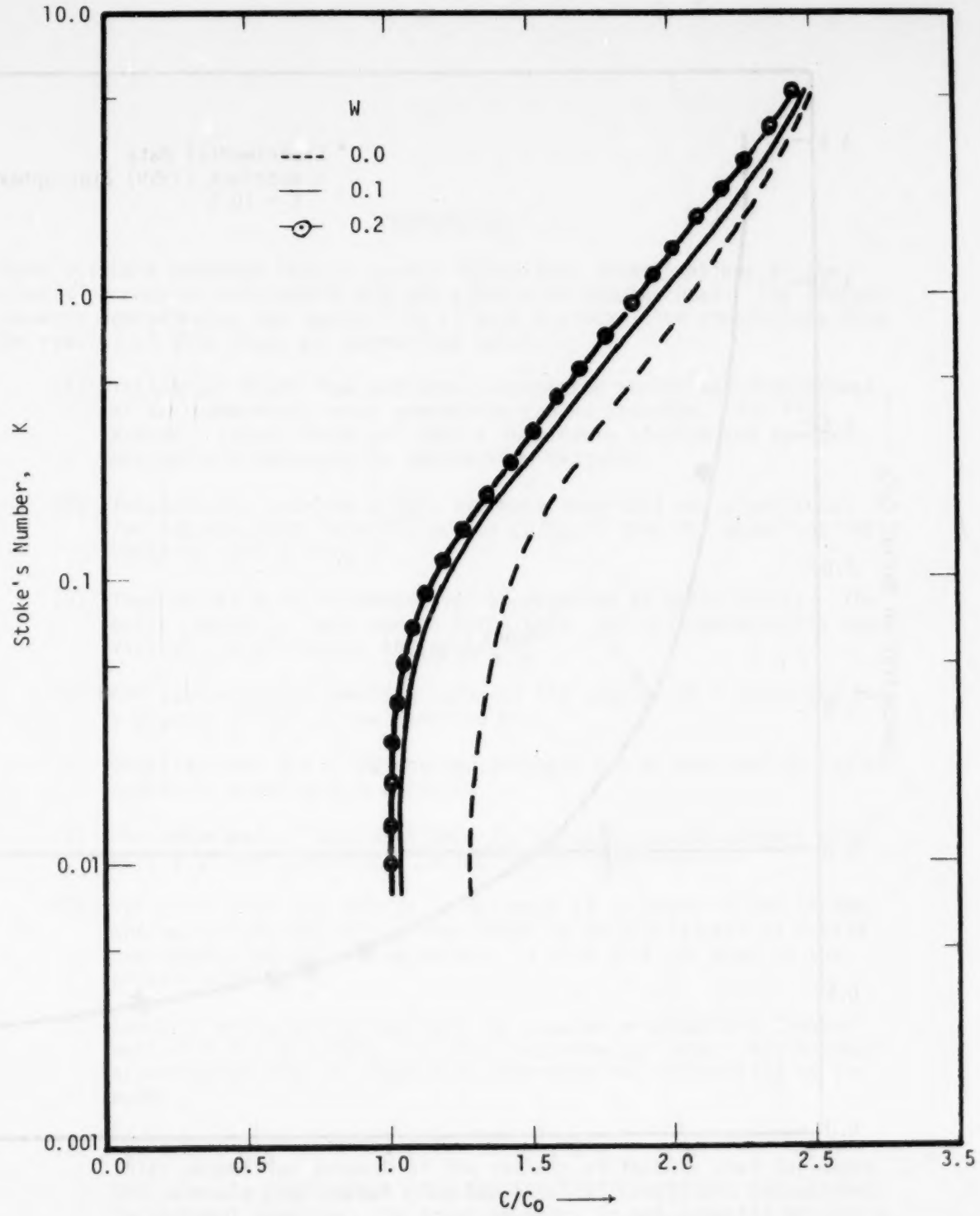


Figure 35. C/C_0 vs. K for Various Thickness of Wall with Circular Inlet Facing the Stream $U/U_0 = 0.375$

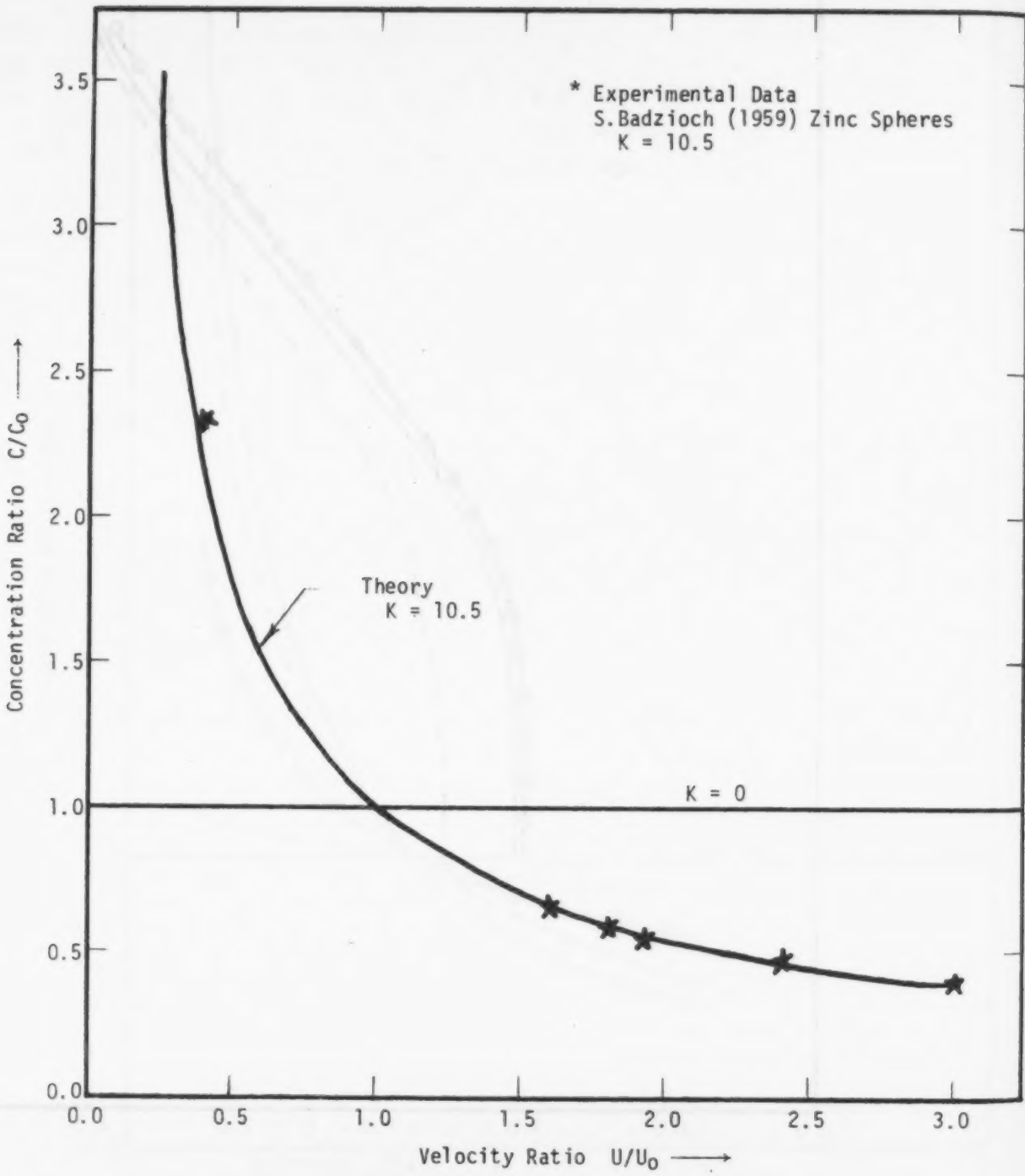


Figure 36. Comparison of Theoretical Results with Experimental Data of S. Badzioch (1959). Circular Tube Facing the Stream.

CONCLUSIONS

There exists a complete lack of purely theoretical investigations on the effects of sampler orientation and the effects of sampler head. The present research demonstrates the feasibility of such a study. The conclusions from the results of this study are summarized below.

(1) The actual fluid flow patterns encountered around two-dimensional or axisymmetrical inlet geometries can be obtained. The flow around a square inlet presents a formidable problem and approximations are necessary to obtain flow patterns.

(2) Anisokinetic sampling errors are more important and significant for subisokinetic velocity ratios ($U/U_0<1$) than for superisokinetic velocity ratios ($U/U_0>1$).

(3) Theoretical error estimates may be regarded as upper limits. The actual error is lower due to turbulence, particle bounce-off, and variability of drag on the particles.

(4) For subisokinetic sampling ($U/U_0<1$) the sampler wall thickness has a counter effect on the sampling bias.

(5) Sampling bias for polydispersed aerosols can be obtained by use of number of monodisperse aerosols.

(6) The experimental data available for circular inlets compare very well with the theoretical collection efficiencies.

(7) For cases when the sampler is oriented at an angle to the stream, the exact solution to the flow field is very difficult to obtain and approximations are necessary. A line sink was used in the present model.

(8) Sampling efficiencies obtained for angular orientations compare well with the presently available experimental data. But expanded experimental data are needed to ascertain the reliability of the model.

(9) It is very difficult to arrive at a single or a multiple optimum inlet geometries because of the variety of factors that influence the particle collection. For the physical conditions encountered in personal sampling, the inlet geometry is not expected to play a major role in influencing the collection efficiency.

(10) The major difficulty in evaluating various inlet geometries is obtaining the flow field around the sampler. Experimentally determined flow field can be used with the particle motion part of the program to accurately evaluate various inlets.

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Appendix A. Derivation of Boundary Condition at Section II

Circular Tube

Using continuity of flow at Sections I and II

$$\frac{1}{2} U_I R_o^2 - \frac{1}{2} UR^2 = \frac{1}{2} U_o R_o^2 - \frac{1}{2} U_o (R + W)^2$$

so

$$U_o = \frac{U_I R_o^2 - UR^2}{R_o^2 - (R + W)^2}$$

denoting the stream function outside the probe at Section II ψ_{op} then

$$\psi_{op} = \frac{1}{2} U_o Y^2 + C$$

on the probe wall ψ is unique and ψ_{op} must be equal to ψ_{Ip}, the stream function inside the probe. So

$$\frac{1}{2} \frac{U_I R_o^2 - UR^2}{R_o^2 - (R + W)^2} (R + W)^2 + C = \frac{1}{2} UR^2$$

and C is given by

$$C = -\frac{1}{2} U_o (R + W)^2 + \frac{1}{2} UR^2$$

$\therefore$

$$\psi_{op} = \frac{1}{2} \frac{U_I R_o^2 - UR^2}{[R_o^2 - (R + W)^2]} \cdot [r^2 - (R + W)^2] + \frac{1}{2} UR^2$$

$$(R + W) \leq r \leq R_o$$

in non-dimensional form

$$\psi_{op} = \frac{R_o^2 - U}{[R_o^2 - (1 + W)^2]} \cdot [r^2 - (1 + W)^2] + U$$

$$(1 + W) \leq r \leq R_o$$

Two Parallel Plates

Following similar procedure as before

$$U_o = \frac{U_I Y_o - UH}{Y_o - (H + W)}$$

$$\psi_{op} = \frac{U_I Y_o - UH}{[Y_o - (H + W)]} \cdot [y - (H + W)] + UH$$

$$(H + W) \leq y \leq Y_o$$

In non-dimensional form

$$\psi_{op} = \frac{Y_o - U}{[Y_o - (1 + W)]} \cdot [y - (1 + W)] + U$$

$$(1 + W) \leq y \leq Y_o$$

Appendix B. Uniform Sink Strength Distribution

$$\psi_{sink} = -\int_{o}^{d} m \cdot dx_1 \cdot \theta$$

$$x_1 = x - y \cot \theta$$

$$dx_1 = y \operatorname{cosec}^2\theta \cdot d\theta$$

$$\psi_{sink} = -\int_{\theta_1}^{\theta_2} m \cdot \theta \cdot \operatorname{cosec}^2\theta \cdot d\theta \cdot y$$

$$= -my \left[\theta_1 \cdot \cot\theta_1 - \theta_2 \cdot \cot\theta_2 + \ell n \frac{\sin\theta_2}{\sin\theta_1}\right]$$

$$\psi_{sink} = -m \left[\tan^{-1}\frac{y}{x} \cdot x - \tan^{-1}\frac{y}{x-d} \cdot (x-d) + y \cdot \ell n \frac{\sqrt{x^2+y^2}}{\sqrt{(x-d)^2+y^2}}\right] \quad (B1)$$

$\psi_{U\cdot F}$ = ψ due to uniform stream

$$= U_I y \cos\alpha + U_I x \sin\alpha$$

$$\psi = \psi_{sink} + \psi_{U\cdot F}$$

$$U_x = \frac{\partial\psi}{\partial y}$$

$$U_x = U_I \cos\alpha - m \ell n \frac{\sqrt{x^2+y^2}}{\sqrt{(x-d)^2+y^2}} \quad (B2)$$

$$U_y = -\frac{\partial\psi}{\partial x}$$

$$= -U_I \sin\alpha + m\left[-\frac{xy}{x^2+y^2} + \tan^{-1}\frac{y}{x} + \frac{(x-d)y}{(x-d)^2+y^2} - \tan^{-1}\frac{y}{(x-d)} + \frac{xy}{x+y} - \frac{(x-d)y}{(x-d)^2+y^2}\right]$$

$$= -U_I \sin\alpha + m\left[\tan^{-1}\frac{y}{x} - \tan^{-1}\frac{y}{x-d}\right] \quad (B3)$$

$$= U_I \sin\alpha + m(\theta_1 - \theta_2)$$

The sink strength m is given as follows

$$H \sin\alpha \cdot U_I + \pi dm = H \cdot U$$

$$\therefore \quad m = \frac{U - \sin\alpha}{2\pi \cdot f} \tag{B4}$$

where $f = \frac{d}{2H}$

Appendix C. Triangular Sink Strength Distribution

$$m_\xi = m \cdot \xi \qquad 0 \le \xi \le \frac{d}{2}$$

$$= m\,(d-\xi) \qquad \frac{d}{2} \le \xi \le d \tag{C1}$$

$$\therefore \quad \psi_{sink} = \int_0^d m_\xi \cdot \theta \cdot d_\xi$$

$$\theta = \tan^{-1}\frac{y}{(x-\xi)} \quad \text{or} \quad \theta = \cot^{-}\frac{(x-\xi)}{y}$$

$$\psi \text{ source} = \int_0^{d/2} m\xi \cot^{-1}\frac{(x-\xi)}{y} \cdot d\xi + \int_{d/2}^{d} m(d-\xi) \cdot \cot^{-1}\frac{(x-\xi)}{y} \cdot d\xi$$

put

$$x = \frac{(x-\xi)}{y}$$

then

$$d\xi = -y \cdot dx$$

then

$$\psi \text{ source} = myx \int_{\frac{x-\frac{d}{2}}{y}}^{\frac{x}{y}} \cot^{-1}x \cdot dx + my^2 \int_{\frac{x}{y}}^{\frac{x-\frac{d}{2}}{y}} x \cdot \cot^{-1}x \cdot dx$$

$$+ my(d-x) \int_{\frac{x-d}{y}}^{\frac{x-\frac{d}{2}}{y}} \cot^{-1}x \cdot dx - my^2 \int_{\frac{x-\frac{d}{2}}{y}}^{\frac{x-d}{y}} x \cdot \cot^{-1}x \cdot dx$$

Integrating and grouping the terms

$$\psi \text{ source} = \cot^{-1}\frac{x}{y}\left[\frac{mx^2}{2} - \frac{my^2}{2}\right]$$

$$+ \cot^{-1}\frac{x-\frac{d}{2}}{y}\left[my^2 - m\left(x-\frac{d}{2}\right)^2\right]$$

$$+ \cot^{-1}\frac{x-d}{y}\left[\frac{m\cdot(d-x)^2}{2} - \frac{my^2}{2}\right]$$

$$+ \frac{mxy}{2}\,\ell n\,\frac{y^2+x^2}{y^2+\left(\frac{x-d}{2}\right)^2}$$

$$+ \frac{my(d-x)}{2}\,\ell n\left[\frac{y^2+\left(x-\frac{d}{2}\right)^2}{y^2+(x-d)^2}\right] \qquad (C2)$$

The total sink strength $= \int_0^{\frac{d}{2}} m \cdot \xi \cdot d\xi + \int_{\frac{d}{2}}^{d} m(d-\xi) \cdot d\xi$

$$= \frac{md^2}{8} + \frac{md^2}{8}$$

$$= \frac{md^2}{4}$$

The value of m is found by flow balance to the sampler. Let Q be the rate of sampling for a square inlet (H x H). Then flow per unit width is $\frac{Q}{H}$ and

$$\frac{Q}{H} = U_I \sin\alpha \cdot H - \pi \cdot \frac{md^2}{4}$$

non-dimensional $\quad m = \frac{mH}{U_I} = -\frac{2(U - \sin\alpha)}{\pi f^2} \qquad (C3)$

where f is given by $\frac{d}{2H}$

velocity $U_x = \frac{\partial\psi}{\partial y} = \frac{\partial\psi_{source}}{\partial y} + \frac{\partial\psi_{U\cdot F}}{\partial y}$

$$U_y = -\frac{\partial\psi}{\partial x} = \frac{\partial\psi_{source}}{\partial x} - \frac{\partial\psi_{U\cdot F}}{\partial x}$$

Appendix D. Computer Program Listing

(See following pages.)

```
C  ***********************************************************************
C
C            'FLOWFI' DESCRIPTION
C           **********************
C
C  ***********************************************************************
C   THIS PROGRAM SOLVES FOR THE FLOW FIELD IN AND AROUND THE SAMPLING
C   HEAD.
C   THE PHYSICAL DIMENSIONS ARE NORMALIZED WITH RESPECT TO THE
C   PROBE RADIUS
C   ALL THE VELOCITIES ARE NORMALIZED WITH RESPECT TO THE FREE
C   STREAM VELOCITY
C
C   METHOD OF SOLUTION
C
C   SECOND ORDER FINITE DIFFERENCE APPROXIMATION
C   SUCCESSIVE OVER RELAXATION(SOR) TECHNIQUE IS USED
C
C   THE FLUID IS ASSUMED TO BE FRICTIONLESS AND THE FLOW FIELD IS GOVERNED
C   BY POTENTIAL FLOW EQUATIONS.THE FLUID FLOW MODEL COVERS THREE DIFFERENT
C   SITUATIONS.1) TWO DIMENSIONAL FLOW BETWEEN PARALLEL PLATES
C   2)AXISYMMETRIC FLOW IN A CIRCULAR TUBE
C   3)LINE SINK WITH ARBITRARY ORIENTATION TO THE ONCOMING FLOW.
C   THE USER CAN CHOOSE ANY OF THE ABOVE OPTION WITH PROPER VALUE OF AN
C   INTEGER PARAMETER 'NDIM'
C     NDIM =0 TWO DIMENSIONAL CASE
C     NDIM =1 AXISYMMETRIC CASE
C     NDIM =2 LINE SINK
C
C  ***********************************************************************
C            'FLOEFI' VARIABLES DESCRIPTION
C           ********************************
C
C   RELAX*****RELAXATION PARAMETER(USER INPUT).VALUES BETWEEN 1.0 AND 2.0
C
C   ALPHA*****ANGLE OF ORIENTATION OF SAMPLER HEAD(INPUT).USED ONLY WITH
C   NDIM VALUE 2.
C   IGR*******GRID POINTS PER UNIT LENGTH IN AXIAL DIRECTION (INPUT)
C
C   JGR*******GRID POINTS PER UNIT LENGTH IN TRANSVERSE DIRECTION(INPUT)
C
C   ZM *******FLOW FIELD BOUNDARY UPSTREAM(INPUT) <=5.
C
C   ZMA*******FLOW FIELD BOUNDARY DOWNSTREAM(INPUT)
C
C   RO *******FLOW FIELD BOUNDARY RADIAL DIRECTION(INPUT)
C
C   ITERMA****MAXIMUM ALLOWABLE ITERATIONS FOR FLOW FIELD TO CONVERGE(INPUT)
C
C   IM  ******MAXIMUM NUMBER OF GRID POINTS IN AXIAL DIRECTION <101
C
C   JM  ******MAXIMUM NUMBER OF GRID POINTS IN RADIAL DIRECTION <61
C
C   ANG ******TAPERING ANGLE OF SAMPLER WALL(INPUT)
C
C   ERR ******MAXIMUM ERROR ALLOWABLE IN STREAM FUNCTION SOLUTION(INPUT)
C
C   ITER******NUMBER OF ITERATIONS TO OBTAIN THE SOLUTION
C
C   PSI*******IMXJM ARRAY OF STREAM FUNCTION
```


best copy available

```
C
C    IPUNCH****CONTROL PARAMETER FOR GETTING PUNCHED OUTPUT OF
C              PSI,UZ,UR,Z,R,ETC(INPUT)
C              =0 NO PUNCHED OUTPUT DESIRED
C              =1 PUNCHED OUTPUT RESULTS
C
C    NCON******INTEGER PARAMETER INDICATING WHETHER SOLUTION CONVERGED
C              OR NOT WITHIN ITERMA INTERATIONS
C              =0 CONVERGENCE OBTAINED
C              =1 NO CONVERGENCE
C
C    U  *******SAMPLING VELOCITY RATIO (INPUT)
C
C    UR********VELOCITY OF FLUID IN TRANSVERSE DIRECTION(ARRAY OF IMXJM)
C
C    UZ********VELOCITY OF FLUID IN AXIAL DIRECTION(ARRAY OF IMXJM)
C
C    W*********THE SAMPLER WALL THICKNESS(INPUT),USED ONLY WITH
C    NDIM OPTION OF 0 OR 1
C
C    ESC*******FUNCTION FOR  EQUALLY SPACED CENTRAL DIFFERENCE
C
C    ESF*******FUNCTION FOR  EQUALLY SPACED FORWARD DIFFERENCE
C
C    ESB*******FUNCTION FOR  EQUALLY SPACED BACKWARD DIFFERENCE
C
C    UESC******FUNCTION FOR  UNEQUALLY SPACED CENTRAL DIFFERENCE
C
C    UESF******FUNCTION FOR  UNEQUALLY SPACED FORWARD DIFFERENCE
C
C    UESB******FUNCTION FOR UNEQUALLY  SPACED  BACKWARD DIFFERENCE
C
C    M*********FICTICIOUS SOURCE/SINK STRENGTH(FOR NDIM 2)
C
C    ***********************************************************************
C                        SUBROUTINE  DESCRIPTION
C    ***********************************************************************
C
C    MAIN******CALLS VARIOUS SUBRCUTINES
C
C    FLBOUN****FIXES THE FLOW BOUNDARY
C
C    GRID******LAYS A PRESPECIFIED GRID ON THE FLOW FIELD AND
C              CALCULATES THE COORDINATES OF THE GRID POINTS.
C
C    BCOND*****CALCULATES BOUNDARY CONDITIONS FOR THE PROBLEM
C
C    LAPLA*****SOLVES THE FINITE DIFFERENCE EQUATION BY SOR METHOD
C
C    VELO******CALCULATES THE VELOCITIES FROM STREAM FUNCTION
C
C    STREM*****CALCULATES THE CONTOUR OF A STREAM LINE
C
C    RESULT****PRINTS  THE  RESULTS
C
C    SINVEL****CALCULATES THE FLOW FIELD WITH LINE SINK(NDIM=2)
C
C
C    ***********************************************************************
C***********************************************************************
```

```
C                          'FLOWFI' DATA INPUT
C                           ******************
C   CARD 1:   NDIM,IPUNCH   FORMAT( )
C
C             NDIM.....FLOW FIELD OPTION PARAMETER
C                      =0 TWO DIMENSIONAL CASE
C                      =1 AXISYMMETRIC CASE
C                      =2 LINE SINK
C
C             IPUNCH...CONTROL PARAMETER FOR GETTING PUNCHED OUTPUT OF
C                      PSI,UZ,UR,Z,R,ERC
C                      =0 NO PUNCHED OUTPUT DESIRED.
C                      =1 PUNCHED OUTPUT DESIRED.
C
C   CARD 2:   ISINK,ALPHA,F   FORMAT(I5,2F10.4)    OMIT IF NDIM.NE.2
C
C             ISINK....SINK DISTRIBUTION PARAMETER
C                      =0 LINE SINK OF UNIFORM STRENGTH.
C                      =1 LINE SINK OF TRIANGULAR STRENGTH DISTRIBUTION.
C
C             ALPHA....ANGULAR ORIENTATION OF THE FACE PLANE OF SAMPLER HEAD.
C                      =90 DEGREES WHEN SAMPLER FACES THE STREAM
C                      =0  DEGREES WHEN STREAM IS TANGENTIAL TO THE HEAD.
C             F........FRACTION OF THE PROBE DIAMETER OVER WHICH
C                      THE SINK IS ASSUMED TO BE DISTRIBUTED.
C                      RECOMMENDED VALUE IS 0.01.
C
C   CARD 3:   ITERMA,RELAX   FORMAT(I5,F10.4)
C
C             ITERMA...MAXIMUM ALLOWABLE ITERATIONS FOR FLOW FIELD
C                      TO CONVERGE
C
C             RELAX....RELAXATION PARAMETER USED IN THE PROCEDURE OF
C                      SUCCESSIVE OVER RELAXATION(SOR)
C                      VALUES BETWEEN 1.0 AND 2.0.
C
C   CARD 4:   ZM,ZMA,RO,U,W,ANG      FORMAT(6F10.4)
C
C             ZM.......UPSTREAM FLOW FIELD BOUNDARY.LE.-5.
C
C             ZMA......DOWNSTREAM FLOW FIELD BOUNDARY
C
C             RO.......FLOW FIELD BOUNDARY IN THE RADIAL OR TRANSVERSE
C                      DIRECTION.  RO.GE.5.
C
C             U........ANISOKINETIC VELOCITY RATIO.
C
C             W........SAMPLER WALL THICKNESS
C                      WHEN NDIM.EQ.2 W=0.
C
C             ANG......TAPERING ANGLE OF SAMPLER WALL
C                      =0 IF W=0. OR NDIM.EQ.2
C
C   CARD 5:   IGZ,JGR    FORMAT(2I5)
C
C             IGZ......GRID POINTS PER UNIT LINGTH IN AXIAL DIRECTION
C                      = 1./GRID SIZE +1
C
C             JGR......GRID POINTS PER UNIT LINGTH IN RADIAL DIRECTION
C                      =1./RADIAL GRID SIZE +1
```

```
C
C    CARD 6:    II        FORMAT( )
C
C               II........NUMBER OF STREAM FUNCTION CONTOURS NEEDED.LE.10
C
C    CARD 7:    STR       FORMAT( )
C
C               STR.......STREAM FUNCTION VALUES FOR THE CONTOUR.THIS ARRAY
C                         WILL CONTAIN 'II' VALUES
C
C    ***********************************************************************
C
C
      PROGRAM MAIN
      DIMENSION PSI(101,61),Z(101),R(61)
      DIMENSION UR(101,61),UZ(101,61)
      DIMENSION STR(10)
      COMMON/AND/NDIM,IPUNCH
      COMMON/BAT/ALPHA,F,ISINK
      READ(5,3) NDIM,IPUNCH
    3 FORMAT( )
      IF(NDIM.EQ.2) READ(5,1) ISINK,ALPHA,F
      READ(5,1) ITERMA,RELAX
    1 FORMAT(I5,2F10.4)
      CALL FLBOUN(ZM,ZMA,RO,U,W,ZOW,ITW)
      CALL GRID(IM,JM,Z,R,ZM,ZMA,RO,W,ZOW,ITW)
      IF(NDIM.EQ.2) GO TO 2
      CALL BCOND(PSI,IM,JM,U,ITW,W,R)
      CALL LAPLA(PSI,IM,JM,ITW,RELAX,Z,R,ITERMA,NCON)
      CALL VELO(PSI,IM,JM,UZ,UR,RO,W,ITW,U,Z,R)
      CALL RESULT(PSI,IM,JM,Z,R,RELAX,NCON,UZ,UR)
      READ(5,3) II
      READ(5,3) (STR(I),I=1,II)
      CALL STREAM(Z,PSI,R,IM,JM,STR,II)
      STOP
    2 CALL SINVEL(U,Z,R,IM,JM,UZ,UR)
      CALL RESULT(PSI,IM,JM,Z,R,RELAX,NCON,UZ,UR)
      STOP
      END
#FOR,I FLBOUN
      SUBROUTINE FLBOUN(ZM,ZMA,RO,U,W,ZOW,ITW)
      COMMON/AND/NDIM,IPUNCH
      COMMON/BAT/ALPHA,F,ISINK
    1 FORMAT(6F10.4)
      READ(5,1)ZM,ZMA,RO,U,W,ANG
      EPANG=ANG-90.
      ITW=1
      IF(ABS(EPANG).LE..005)ITW=1
      IF(ANG.LE..005)ITW=0
      AN =ANG/180.*3.1415962
      IF(ITW)2,3,3
    3 ZOW=0.
      GO TO 4
    2 ZOW=W/TAN(AN )
    4 WRITE(6,5)
    5 FORMAT(1H1,25X,'FLOW FIELD BOUNDARY'///)
      IF(NDIM.EQ.0) WRITE(6,8)
      IF(NDIM.EQ.1) WRITE(6,9)
      IF(NDIM.EQ.2) WRITE(6,10) ALPHA
    8 FORMAT(1H,10X,'FLOW FIELD FOR TWO PARALLEL PLATES'///,10X,
```

```
     1'TWO DIMENSIONAL CASE'///)
    9 FORMAT(1H,10X,'FLOW FIELD FOR CIRCULAR TUBE',///,10X,
     1'AXIALLY SYMMETRIC CASE'///)
   10 FORMAT(1H,9X,'FLOW FIELD WITH A LINE SINK',///,10X,
     1'ORIENTATION ANGLE ALPHA=',F10.6,'DEGREES TO THE FLOW')
      WRITE(6,6) ZM,ZMA,RO
    6 FORMAT(1H,'DISTANCE OF UPSTREAM BOUNDARY=',F10.5//,
     1       1H,'DISTANCE OF DOWN STREAM BOUNDARY=',F10.5//,
     2       1H,'DISTANCE OF RADIAL BOUNDARY=',F10.5//)
      IF(IPUNCH.EQ.1) WRITE(1,1)U,W,ANG
      WRITE(6,7) U,W,ANG
    7 FORMAT(1H,'SAMPLING CONDITIONS'////,
     1       1H,'SAMPLING VELOCITY/FREE STREAM VELOCITY=',F10.5//,
     2       1H,'SAMPLER WALL THICKNESS=',F10.5//,
     3 1H,' TAPERING ANGLE OF SAMPLER WALL=',F10.5,'DEGREES'//)
      RETURN
      END
#FOR,I GRID
      SUBROUTINE GRID(IM,JM,Z,R,ZM,ZMA,RO,W,ZOW,ITW)
      COMMON/ABC/JGR,IGR,IBW,ITER
      COMMON/ACC/IBWO,JGRO
      DIMENSION Z(101),R(61)
      READ(5,18) IGZ,JGR
   18 FORMAT(2I5)
      IGR=IGZ
      WRITE(6,1) IGR,JGR
    1 FORMAT(1H,'NUMBER OF GRID POINTS FOR UNIT DISTANCE Z=',I5//,
     1       1H,'NUMBER OF GRID POINTS FOR UNIT DISTANCE R=',I5//)
      FGZ=IGZ-1
      FGR=JGR-1
      DZ=1./FGZ
      DR=1./FGR
C     CALCULATE MAXIMUM VALUE OF I
      I=1
      Z(I)=ZM
    2 Z(I+1)=Z(I)+DZ
      I=I+1
      IF(ABS(Z(I)).LE.(DZ+.0005))GO TO 3
      GO TO 2
    3 I=I+1
      Z(I)=0.
      IBW=I
      IF(ITW)5,15,15
    5 FN=ZOW/DZ
      N=FN
      NN=FN+.99
      IF(NN.EQ.N) GO TO 13
      FN=NN
      N=NN
   13 DO 14 J=1,N
      FJ=J
      I=I+1
      Z(I)=FJ*ZOW/FN
   14 CONTINUE
      Z(I)=ZOW
   15 IBWO=I
    4 Z(I+1)=Z(I)+DZ
      I=I+1
      IF(Z(I).GE.(ZMA-.0005))GO TO 6
      GO TO 4
```

```
      6 IM=1
C       CALCULATE MAXIMUM VALUE OF J
        J=1
        R(J)=0.
      7 R(J+1)=R(J)+DR
        J=J+1
        IF(J.EQ.JGR)GO TO 8
        GO TO 7
      8 R(JGR)=1.
        IF(ITW)9,10,17
     17 FN=W/DR
        N=FN
        NN=FN+.99
        IF(NN.EQ.N) GO TO 9
        FN=NN
        N=NN
      9 DO 16 JJ=1,N
        FJ=JJ
        J=J+1
        R(J)=1.+FJ*W/FN
     16 CONTINUE
        R(J)=1.+W
        JGRO=J
     10 J=J+1
        R(J)=R(J-1)+DR
        IF((R(J)+.005).GE.RO)GO TO 11
        GO TO 10
     11 JM=J
        IF(ITW.EQ.0) JGRO=JGR
        WRITE(6,12) IM,JM
     12 FORMAT(1H,'MAXIMUM NUMBER OF POINTS IN Z DIRECTION=',I5//,
       1       1H,'MAXIMUM NUMBER OF POINTS IN R DIRECTION=',I5//)
        RETURN
        END
#FOR,I BCOND
        SUBROUTINE BCOND(PSI,IM,JM,U,ITW,W,R)
        COMMON/ABC/JGR,IGR,IBW,ITER
        COMMON/ACC/IBWO,JGRO
        COMMON/ARD/NDIM,IPUNCH
        DIMENSION PSI(IM,JM),R(JM)
C       INITIALIZE ALL PSI VALUES.
        N=NDIM+1
        DO 1 1 J=1,JM
        A=R(J)**N
        DO 1 I=1,IM
        PSI(I,J)=A
      1 CONTINUE
     11 CONTINUE
C       AT THE OUTER BOUNDARY
        DO 2 I=1,IM
        PSI(I,JM)=R(JM)**N
      2 CONTINUE
C       AT THE INLET
        JM1=JM-1
        DO 3 J=2,JM1
        PSI(1,J)=R(J)**N
      3 CONTINUE
C       AT THE OUTLET
        DO 4 J=2,JGR
        PSI(IM,J)=R(J)**N*U
```

```
    4 CONTINUE
      DO 5 I=IBW,IM
      PSI(I,JGR)=0
    5 CONTINUE
      JG=JGR+ABS(ITW)
      IF(ITW)6,8,6
    6 IH=IHW
      IF(ITW.LE.0)IH=IBW+1
      DO 7 I=IB,IBW0
      JGG=JGR-IHW+I
      IF(ITW.EQ.1) JGG=JGRO
      DO 13 J=JG,JGG
      PSI(I,J)=PSI(I,JGR)
   13 CONTINUE
    7 CONTINUE
      DO 14 I=IBW0,IM
      DO 15 J=JG,JGG
      PSI(I,J)=PSI(I,JGR)
   15 CONTINUE
   14 CONTINUE
    8 A=R(JM)**N-1
      C=(1.+W)**N
      B=R(JM)**N-C
      IF(ITW.EQ.0) JGG=JG
      DO 9 J=JGG,JM1
      IF(J.EQ.JGR)GO TO 9
      PSI(IM,J)=A/B*(R(J)**N-C)+1
    9 CONTINUE
      WRITE(6,12)
   12 FORMAT(1H,'INITIAL VALUES OF STREAM FUNCTION'//)
      WRITE(6,10)((PSI(I,J),I=1,IM,5),J=1,JM,5)
   10 FORMAT(11F10.4)
      RETURN
      END
*FOR,I LAPLA
      SUBROUTINE LAPLA(PSI,IM,JM,ITW,RELAX,Z,R,ITERMA,NCON)
      COMMON/ABC/JGR,IGR,IHW,ITER
      COMMON/AAA/ERR
      COMMON/ACC/IBW0,JGRO
      COMMON/ABC/NDIM,IPUNCH
      DIMENSION PSI(IM,JM),Z(IM),R(JM)
      NDIM=NDIM
      WRITE(6,11) RELAX
   11 FORMAT(1H,'RELAXATION FACTOR=',F15.6,//,1H,
     1'CONVERGENCE RATE'//)
      IG=0
      NCON=0
      ITER=1
      IM1=IM-1
      JM1=JM-1
      EPPS=1E-5
      IF(ITW.NE.0)IG=1
    3 ERR=0.
      DO 1 I=2,IM1
      DZ=Z(I)-Z(I-1)
      IF(IG.EQ.0)GO TO 8
      DZZ=Z(I+1)-Z(I)
      A=2./(DZ+DZZ)
      B=A/DZ
      C=A/DZZ
```

```
      D=2./DZ/DZZ
    4 DO 2 J=2,JM1
      DR=R(J)-R(J-1)
      IF(J.EQ.JGR.AND.I.GE.IBR) GO TO 2
      IF(IG.EQ.0)GO TO 5
      IF(J.LT.JGR.OR .J.GT.JGRO) GO TO 10
      IF(I.LT.IBR) GO TO 10
      IF(ITR)9,5,2
    9 IF(I.GE.IBRO) GO TO 2
      JMAG=I-IBR+JGR+1
      IF(J.GE.JMAG) GO TO 10
      GO TO 2
   10 DRR=R(J+1)-R(J)
      A=2./(DR+DRR)
      B=A/DR
      F=A/DRR
      G=B/2.*DRR/R(J)*FNDIM
      H=F/2.*DR/R(J)
      H=H*FNDIM
      P=(DR-DRR)/DR/DRR/R(J)
      P=P*FNDIM
      Q=2./DR/DRR
      S=B-P+Q
      PS=PSI(I,J)+RELAX/S*(B*PSI(I-1,J)+C*PSI(I+1,J)+(G+E)*PSI(I,J-1)+(F
     1-H)*PSI(I,J+1))
      PS=PS-RELAX*PSI(I,J)
      GO TO 6
    5 PS=PSI(I,J)+RELAX/4.*(PSI(I-1,J)+PSI(I+1,J)+PSI(I,J-1)*(1.+DR/2.*
     2FNDIM/R(J))+PSI(I,J+1)*(1.-DR/2./R(J)*FNDIM))
      PS=PS-RELAX*PSI(I,J)
    6 ER=ABS(PSI(I,J)-PS)
      FR=ER/PSI(I,J)
      IF(ER.GE.ERR)ERR=ER
      PSI(I,J)=PS
    2 CONTINUE
    1 CONTINUE
      WRITE(6,8) ITER,ERR
    8 FORMAT(I5,F10.7)
      IF(ERR.LE.EPPS)GO TO 7
      ITER=ITER+1
      IF(ITER.GE.ITERMA)NCON=1
      IF(ITER.GE.ITERMA)GO TO 7
      GO TO 3
    7 RETURN
      END
$FOR,I RESULT
      SUBROUTINE RESULT(PSI,IM,JM,Z,R,RELAX,NCON,UZ,UR)
      DIMENSION PSI(IM,JM),Z(IM),R(JM)
      DIMENSION UZ(IM,JM),UR(IM,JM)
      COMMON/ABC/JGR,IGR,IBR,ITER
      COMMON/AAA/ERR
      COMMON/AND/NDIM,IPUNCH
      COMMON/ACC/IBRO,JGRO
      COMMON/HAT/ALPHA,F,ISINK
      IF(NDIM.EQ.2) GO TO 12
      IF(NCON)1,1,2
    2 WRITE(6,3)
    3 FORMAT(1H,'NO CONVERGENCE')
    1 WRITE(6,5)ITER
    5 FORMAT(1H,'NUMBER OF ITERATIONS',2X,I8)
```

```
      WRITE(6,4) ERR
    4 FORMAT(1H,'MAXIMUM ERROR=',2X,F10.7)
      WRITE(6,7)
    7 FORMAT(//,1H,' FINAL STREAM FUNCTION SOLUTION '//)
      WRITE(6,6)((PSI(I,J),I=1,IM,5),J=1,JM)
   12 WRITE(6,8)
      WRITE(6,6)(( UZ(I,J),I=1,IM,5),J=1,JM)
      WRITE(6,9)
      WRITE(6,6)(( UR(I,J),I=1,IM,5),J=1,JM)
    6 FORMAT(11F10.4)
    8 FORMAT(//,1H,5X,'UZ')
    9 FORMAT(//,1H,5X,'UR')
   10 FORMAT(8F10.4)
   11 FORMAT(2I4)
      IF(IPUNCH)13,13,14
   14 CONTINUE
      WRITE(1,11) NDIM
      WRITE(1,11) JGR,JGRO
      WRITE(1,11) IBW,IBWO
      WRITE(1,11) IM,JM
      IF(NDIM.EQ.2) WRITE(1,11) ISINK
      IF(NDIM.EQ.2) WRITE(1,10) F,ALPHA
      WRITE(1,10)(Z(I),I=1,IM)
      WRITE(1,10)(R(I),I=1,JM)
      IF(NDIM.EQ.2) GO TO 13
      WRITE(1,10)((UZ(I,J),I=1,IM),J=1,JM)
      WRITE(1,10)((UR(I,J),I=1,IM),J=1,JM)
   13 RETURN
      END
*FOR,1 VELO
      SUBROUTINE VELO(PSI,IM,JM,UZ,UR,RO,N,ITW,U,Z,R)
      DIMENSION Z(IM),R(JM)
      DIMENSION PSI(IM,JM),UZ(IM,JM),UR(IM,JM)
      COMMON/ALC/IBWO,JGRO
      COMMON/ABC/JGR,IGR,IBW,ITER
      COMMON/AND/NDIM,IPUNCH
      ESC(A,B,C)=(A-B)/2./C
      ESB(A,B,C,D)=(3.*A+C-4.*B)/2./D
      ESF(A,B,C,D)=(4.*B-A-3.*C)/2./D
      DESC(A,H,C,E,D)=(D** 2*A-(D**2-E**2)*H-E**2*C)/(D*E)/D/E
      DFSF(A,H,C,D,E)=(R*(D+E)**2-A*D**2-C*(2.*D+E)*E)/D/E/(D+E)
      DFSB(A,H,C,D,E)=(A*(2.*D+E)*E    -H*(D+E)**2+C*D**2)/D/E/(D+E)
C INITIALIZE ALL VALUES OF VELOCITIES
      N=NDIM+1
      DO 1 I=1,IM
      DO 2 J=1,JM
      UZ(I,J)=1.0
      UR(I,J)=0.0
    2 CONTINUE
    1 CONTINUE
      DO 3 J=1,JGR
      UZ(IM,J)=0
    3 CONTINUE
      DO 4 J=JGRO,JM
      UZ(IM,J)=(RO**N-0)/(RO**N-(1.+W)**N)
    4 CONTINUE
      DO 28 J=JGR,JGRO
      UZ(IM,J)=0.
   28 CONTINUE
      DO 29 I=1,IM
```

```
      UZ(I,1)=PSI(I,2)/R(2)**N
      IF(NDIM.EQ.0) RR=1.
   29 CONTINUE
C CALCULATE THE VELOCITIES
      IM1=IM-1
      JM1=JM-1
      DO 5 I=2,IM1
      DO 7 J=2,JM1
      DZ=Z(I+1)-Z(I)
      RR=2.*R(J)
      IF(NDIM.EQ.0) RR=1.
      DR=R(J+1)-R(J)
      IF(ITW)8,9,10
    9 UZ(I,J)=ESC(PSI(I,J+1),PSI(I,J-1),DR)/RR
      UR(I,J)=ESC(PSI(I+1,J),PSI(I-1,J),DZ)/(-RR)
      GO TO 7
   10 IF(J.LT.JGW.OR.J.GT.JGRO)GO TO 9
      IF(I-IBW)11,12,13
   11 IF(J.GT.JGW.AND.J.LT.JGRO )GO TO 9
   17 DRR=R(J)-R(J-1)
      UR(I,J)=-ESC(PSI(I+1,J),PSI(I-1,J),DZ)/RR
      UZ(I,J)=UESC(PSI(I,J+1),PSI(I,J),PSI(I,J-1),DR,DRR)/RR
      GO TO 7
   12 UR(I,J)=-ESH(PSI(I,J),PSI(I-1,J),PSI(I-2,J),DZ)/RR
      UZ(I,J)=0.0
   14 IF(J.EQ.JGW)UZ(I,J)=ESB(PSI(I,J),PSI(I,J-1),PSI(I,J-2),(R(J)-R(J-1
     1)))/RR
      IF(J.EQ.JGRO)UZ(I,J)=ESF(PSI(I,J+2),PSI(I,J+1),PSI(I,J),DR)/RR
      GO TO 7
   13 UR(I,J)=0.0
      UZ(I,J)=0.0
      GO TO 14
    8 IF(I.GE.IBW.AND.I.LE.IBWO)GO TO 15
      IF(J.EQ.JGW.OR .J.EQ.JGRO)GO TO 16
      IF(I.LT.IBW)GO TO 9
      IF(J.LT.JGW.OR.J.GT.JGRO)GO TO 9
      UZ(I,J)=0.0
      UR(I,J)=0.0
      GO TO 7
   16 IF(I.LT.IBW) GO TO 17
      UR(I,J)=0.0
      GO TO 14
   15 IF(I.EQ.IBW.OR.I.EQ.IBWO) GO TO 18
      IF(J.LT.JGW.OR.J.GT.JGRO) GO TO 9
      IF(J.EQ.JGRO) GO TO 12
      IF(J.EQ.JGW) UR(I,J)=0.0
      IF(J.EQ.JGW) GO TO 14
      IF(I.GT.(IBW+J-JGW)) GO TO 27
      IF(J.GT.(JGW+I-IBW))GO TO 9
      IF(J.EQ.(JGW+1).OR.J.EQ.(JGRO-1))GO TO 19
      IF(J.LT.(JGW+I-IBW)) GO TO 27
      DZ=Z(I)-Z(I-1)
      UR(I,J)=-ESH(PSI(I,J),PSI(I-1,J),PSI(I-2,J),DZ)/RR
   21 UZ(I,J)=ESF(PSI(I,J+2),PSI(I,J+1),PSI(I,J),DR)/RR
      GO TO 7
   27 UZ(I,J)=0.0
      UR(I,J)=0.0
      GO TO 7
   19 IF(J.NE.(JGW+1))GO TO 20
      DZ=Z(I)-Z(I-1)
```

```
      DZZ=Z(I-1)-Z(I-2)
      UR(I,J)=UESH(PSI(I,J),PSI(I-1,J),PSI(I-2,J),DZ,DZZ)/(-RR)
      IF(J.EQ.(JGRO-1))GO TO 20
      GO TO 21
   20 DRR=R(J+2)-R(J+1)
      UZ(I,J)=UESH(PSI(I+2,J),PSI(I+1,J),PSI(I,J),DR,DRR)/RR
      IF(J.EQ.(JGR+1))GO TO 7
      DZ=Z(I)-Z(I-1)
      UR(I,J)=-ESH(PSI(I,J),PSI(I-1,J),PSI(I-2,J),DZ)/RR
      GO TO 7
   18 IF(I.EQ.IRHO)GO TO 22
   26 DRR=R(J)-R(J-1)
      DZZ=Z(I)-Z(I-1)
      UR(I,J)=-UESC(PSI(I+1,J),PSI(I,J),PSI(I-1,J),DZ,DZZ)/RR
      UZ(I,J)=UESC(PSI(I,J+1),PSI(I,J),PSI(I,J-1),DR,DRR)/RR
      GO TO 7
   22 IF(J.GT.JGR.AND.J.LT.JGRO) GO TO 23
      IF(J.EQ.JGR) GO TO 24
      IF(J.EQ.JGRO) GO TO 25
      DR=R(J+1)-R(J)
      GO TO 26
   24 UR(I,J)=0.0
      DZ=Z(I)-Z(I-1)
      UR(I,J)=-ESH(PSI(I,J),PSI(I-1,J),PSI(I-2,J),DZ)/RR
      DR=R(J)-R(J-1)
      UZ(I,J)=ESD(PSI(I,J),PSI(I,J-1),PSI(I,J-2),DR)/RR
      GO TO 7
   25 UR(I,J)=0.0
      DZ=Z(I)-Z(I-1)
      UR(I,J)=-ESH(PSI(I,J),PSI(I-1,J),PSI(I-2,J),DZ)/RR
      UZ(I,J)=ESF(PSI(I,J+2),PSI(I,J+1),PSI(I,J),DR)/RR
      GO TO 7
   23 UR(I,J)=0.0
      UZ(I,J)=0.0
    7 CONTINUE
    5 CONTINUE
      RETURN
      END
#FOR.I STREAM
      SUBROUTINE STREAM(Z,PSI,R,IM,JM,STR,II)
      DIMENSION PSI(IM,JM),Z(IM),R(JM),STR(II)
      DIMENSION RS(10,101)
      SAG(X,Y,V,A,B,C,D)=(A-C)*(A-D)*X/(B-C)/(B-D)+(A-B)*(A-D)*Y/(C-B)
     1/(C-D)+(A-B)*(A-C)*V/(D-B)/(D-C)
      JM2=JM-2
      DO 1 IK=1,II
      ST=STR(IK)
      DO 2 I=1,IM
      DO 3 J=1,JM2
      IF(ST.GT.PSI(I,J).AND.ST.LE.PSI(I,J+1)) GO TO 4
    3 CONTINUE
    4 RS(IK,I)=SAG(R(J),R(J+1),R(J+2),ST,PSI(I,J),PSI(I,J+1),PSI(I,J+2))
    2 CONTINUE
    1 CONTINUE
      WRITE(6,5)((Z(I),(RS(IK,I),IK=1,II)),I=1,IM)
    5 FORMAT(11F10.4)
      RETURN
      END
#FOR.I SINVEL
      SUBROUTINE SINVEL(U,Z,R,IM,JM,UZ,UR)
```

```
      DIMENSION Z(IM),R(JM),UZ(IM,JM),UR(IM,JM)
      COMMON/BAT/ALPHA,F,ISINK
      FUN(X,Y)=SQRT(X**2+Y**2)
      REAL M
      ALPHA=ALPHA*3.141596/180.
      C=COS(ALPHA)
      S=SIN(ALPHA)
      M=(U-S)/4./F
      IF(ISINK.EQ.1) M=(S-U)/2./F**2
      WRITE(6,4) M,F,ISINK
    3 FORMAT(4F10.6)
    4 FORMAT(2F10.6,I4)
      DO 1 I=1,IM
      Y=Z(I)
      DO 2 J=1,JM
      X=R(J)+F
      XF=X-F
      XFF=XF-F
      IF(ISINK.EQ.0) GO TO 13
      IF(ABS(Y).LE..01) GO TO 10
      A=FUN(X,Y)
      B=FUN(XF,Y)
      D=FUN(XFF,Y)
      F=B+XF
      E=XFF+D
      G=X+A
      UUZ=-X/2./A+XF/M-XFF/2./D+ALOG(M/G)+ALOG(F/E)+
     1Y**2/2.*(2./M/B-1./G/A-1./E/D)
      UUR=X**2/2./Y/A-XF**2/Y/B+XFF**2/2./Y/D-Y/2.*(2./B-1./A-1./D)
      UUR=UUR+A/2./Y-B/Y+D/2./Y
      UUZ=UUZ*M
      UUR=UUR*M
      GO TO 11
   10 IF(X.GE.0..AND.X.LE.(2.*F)) GO TO 12
      UUR=0.
      UUZ=M*ALOG(XF**2/X/XFF)
      GO TO 11
   12 UUZ=0.
      UUR=0.
      GO TO 11
   13 A=FUN(X,Y)
      B=FUN(XFF,Y)
      IF(A.LT..01) A=.01
      IF(B.LT..01) B=.01
      UUZ=M*(1./A-1./B)
      UUR=-M*(X/A/Y-(XFF)/B/Y)
   11 CONTINUE
      UZ(I,J)=-UUR+S
      UR(I,J)=UUZ+C
    2 CONTINUE
    1 CONTINUE
      RETURN
      END
$ENTRY
$$
```

```
#FOR.I INER
C    ***********************************************************************
C    ***********************************************************************
***********************************************************************
**********************************************************************
**********************************************************************
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**********************************************************************
C    **********************************************************************
C    TRAJEC SOLVES FOR THE LIMITING TRAJECTORY OF PARTICLES CAPTURED BY
C         A SAMPLING HEAD IN A PREDETERMINED FLOW FIELD
C
C    PARTICLE TRAJECTORIES ARE OBTAINED BY NUMERICALLY SOLVING THE EQUATION
C         OF MOTION OF PARTICLES WITH STOKE'S DRAG.
C
C    NUMERICAL METHOD USED.........PREDICTOR-CORRECTOR WITH ITERATIVE CONVERGENCE
C
C    **********************************************************************
C    PROGRAM TRAJEC VARIABLES DESCRIPTION
C    ALPHA********ORIENTATION ANGLE OF THE SAMPLERHEAD WITH RESPECT TO FREE STREAM
C                 IN DEGREES(USED ONLY WITH THE OPTION NDIM=2)    INPUT
C                 =0 WHEN THE FACE PLANE OF THE SAMPLER IS TANGENTIAL TO
C                 FREE STREAM
C
C                 =90 WHEN THE FACE PLANE FACES THE STREAM.
C
C    ANG*********TAPERING ANGLE OF THE SAMPLER WALL(USED ONLY WITH THE OPTIONS
C                 NDIM=1 OR 0)                                    INPUT
C
C    DTI*********INITIAL TIME INCREAMENT FOR THE TRAJECTORY CALCULATION(DURING
C                 THE EXECUTION OF THE PROGRAM THIS TIME INCREAMENT IS VARIED AS
C                 REQUIRED FOR CONVERGENCE CRITERIA.  (USUALLY=0.1)    INPUT
C    DR**********INCREAMENT IN RADIAL DIRECTION. USED TO CHOOSE A NEW'RI'
C                 BASED ON THE PREVIOUS TRAJECTORY CALCULATION (CAN BE 0.1 OR 0.2)
C
C    EPPS********PRESET TOLERENCE TO STOP THE ITERATIVE PROCEDURE AND TRAJECTORY
C                 CALCULATION                                     (INPUT)
C
C    F***********FRACTION OF DIAMETER OVER WHICH THE SINK IS DISTRIBUTED.
C                 (USED ONLY WITH OPTION NDIM=2) USUALLY F=0.01    INPUT
C
C    FR**********FROUDE NUMBER OF THE PARTICLE (OUTPUT)
C
C    IBW*********NUMBER OF AXIAL GRID POINT AT WHICH THE INNER WALL OF THE
C                 SAMPLER IS LOCATED
C
C    IBWO********NUMBER OF AXIAL GRID POINT AT WHICH THE OUTER WALL OF THE
C                 SAMPLER IS LOCATED.
C                 IBWO=IBW IF OPTION NDIM=2 IS USED.
C
C    IDIM********SINK TYPE PARAMETER
C                 =0 TWO DIMENSIONAL
C                 =1 AXISYMMETRICAL
C
C    IE**********ELECTROSTATIC PARAMETER
C                 =0ELECTROSTATIC EFFECT NOT TAKEN INTO ACCOUNT
C                 =1    "       "       "   TAKEN INTO ACCOUNT
C
C    IM**********MAXIMUM NUMBER OF GRID POINTS IN AXIAL DIRECTION  INPUT
C
```

```
C   INT********INTERVAL FOR TRAJECTORY COORDINATES TO BE PRINTED OUT  INPUT
C
C   IS*********SEDIMENTATION PARAMETER
C              =0 SETTLING IS NOT TAKEN INTO ACCOUNT
C              =1     "    "  TAKEN INTO ACCOUNT
C
C   ISINK*******SINK STRENGTH DISTRIBUTION PARAMETER.
C              =0 UNIFORM STRENGTH
C              =1 TRIANGULAR STRENGTH DISTRIBUTION
C
C   IST*********NUMBER OF SIZE INTERVALS IN PARTICLES DISTRIBUTION  INPUT
C              MAXIMUM OF 10
C
C   LIMIT*******MAXIMUM NUMBER OF TIMES TO PERFORM THE TRAJECTORY CALCULATION
C              FOR ONE PARTICLE
C
C   THE FLUID IS ASSUMED TO BE FRICTIONLESS AND THE FLOW FIELD IS GOVERNED
C   BY POTENTIAL FLOW EQUATIONS.THE FLUID FLOW MODEL COVERS THREE DIFFERENT
C   SITUATIONS.1) TWO DIMENSIONAL FLOW BETWEEN PARALLEL PLATES
C   2)AXISYMMETRIC FLOW IN A CIRCULAR TUBE
C   3)LINE SINK WITH ARBITRARY ORIENTATION TO THE ONCOMING FLOW.
C   THE USER CAN CHOOSE ANY OF THE ABOVE OPTION WITH PROPER VALUE OF AN
C   INTEGER PARAMETER 'NDIM'
C     NDIM =0 TWO DIMENSIONAL CASE
C     NDIM =1 AXISYMMETRIC CASE
C     NDIM =2 LINE SINK
C
C   P**********PARTICLE RADIUS IN MICRONS  (DIMENSION 'IST')
C
C   PF*********MEASURED FRACTION OF VARIOUS PARTICLE SIZES (DIMENSION 'IST')
C
C   PL*********LINEAR DIMENSION OF THE SAMPLING HEAD
C
C   QC*********ELECTROSTATIC CHARGE ON THE SAMPLER HEAD.
C
C   RI*********INITIAL RADIAL POSITION OF THE PARTICLE
C
C   RT*********RADIUS OF THE TUBE(SAMPLING) OR CHARECTERISTIC DIMENSION OF
C              THE SAMPLER.
C
C   STK********STOKE'S NUMBER OF THE PARTICLE
C
C   R**********RADIAL COORDINATE(DIMENSION 'JM')
C
C   RE*********REYNOLD'S NUMBER BASED ON THE RADIUS
C
C   TA*********RELAXATION TIME OF THE PARTICLES (DIMENSION IST)
C
C   W**********THE SAMPLER WALL THICKNESS(INPUT).USED ONLY WITH
C   NDIM OPTION OF 0 OR 1
C
C   UZ*********VELOCITY OF FLUID IN AXIAL DIRECTION(ARRAY OF IMXJM)
C
C   UR*********VELOCITY OF FLUID IN TRANSVERSE DIRECTION(ARRAY OF IMXJM)
C
C   U  ********SAMPLING VELOCITY RATIO (INPUT)
C
C   Z**********AXIAL COORDINATE (DIMENSION IM)
C
C   ZI*********INITIAL AXIAL POSITION OF THE PARTICLE
```

```
C   ZLIP*******LIP DEPTH OF THE SAMPLER
C
C   ZMAX*******MAXIMUM AXIAL DISTANCE UPTO WHICH THE TRAJECTORY SHOULD BE
C              CONTINUED (USUALLY EQUAL TO 0.)
C
C
C***********************************************************************
C
C
C
C
C
C
C
C
C
C
C
C
C                    TRAJEC DATA INPUT
C
C   &&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&
C   &&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&
C   CARD 1:  INT       FORMAT(I4)
C        PRINTING INTERVAL
C
C   CARD 2: IE,IW,IS FORMAT(3I4)
C        IE=0/1  ELECTROSTATIC EFFECT NOT INCLUDED/INCLUDED
C        IW=1/-1  UPPER/LOWER WALL
C        IS=0/1    SEDIMENTATION EFFECT NOT INCLUDED/INCLUDED
C
C   CARD 3: U,W,ANG    FORMAT(3F10.4)
C        U--VELOCITY RATIO,SAMPLING VEL/FREE STREAM VEL
C        W--WALL THICKNESS OF THE SAMPLER
C        ANG--TAPERING ANGLE IN DEGREES OF SAMPLER WALL
C
C   CARD4: NDIM   FORMAT(I4)
C        FLOW FIELD OPTION
C        =0 TWO DIMENSIONAL FLOW BETWEEN PARALLEL PLATES
C        =1 AXISYMMETRICAL FLOW FOR CIRCULAR TUBE
C        =2 FLOW FIELD WITH LINE SINK
C        OPTIONS 0&1 REQUIRE THE FLOW FIELD TO BE DETERMINED BY THE PROGRAM
C        'FLOWFI'
C
C   CARD5: IDIM  FORMAT(I4)   OMIT IF NDIM=0 OR 1
C        =0 TWO DIMENSIONAL LINE SINK
C        AXISYMMETRICAL LINE SINK
C
C   CARD 6: JGR,JGRO              FORMAT(2I4)
C        JGR--NUMBER OF GRID POINTS PER UNIT
C        RADIAL DISTANCE, =1./GRID SPACINGIN RAD.DIR+1
C        JGRO--RADIAL GRID POINT AT WHICH OUTER EDGE OF THE WALL IS LOCATED
C        JGRO=JGR IF NDIM=2 OR W=0. FOR NDIM=0,1 JGR&JGRO ARE CALCULATED
C        BY THE PROGRAM FLOWFI
C
C   CARD 7: IBW,IBWO    FORMAT(2I4)
C        IBW--AXIAL GRID POINT AT WHICH INNER EDGE OF SAMPLER WALL IS LOCATED
C        IBWO--AXIAL GRID POINT AT WHICH OUTER EDGE OF SAMPLER WALL IS
```

```
C       LOCATED. IBWD=IBW IF NDIM=2 OR W=0 OR ANG=90. FOR NDIM=0 OR 1,
C       IBW AND IBWD ARE CALCULATED BY 'FLOWFI'
C
C   CARD 8: IM,JM    FORMAT(2I4)
C       IM--MAXIMUM NUMBER OF POINTS IN AXIAL DIRECTION <101
C       JM--MAXIMUM NUMBER OF POINTS IN RADIAL DIRECTION,<61
C       FOR NDIM=2 IM,JM ARE INPUT BY THE USER.
C       FOR NDIM=0,1 IM,JM ARE OUTPUT BY FLOWFI.
C
C   CARD 9: ISINK   FORMAT(I4)   OMIT IF NDIM.NE.2
C       =0  UNIFORM SINK STRENGTH DISTRIBUTION.
C       =1  TRIANGULAR SINK STRENGTH DISTRIBUTION.
C
C   CARD 10: F,ALPHA   FORMAT(2F10.4)   OMIT IF NDIM.NE.2
C       F--FRACTION OF DIAMETER OVER WHICH THE LINE SINK IS DISTRIBUTED
C       USUALLY F=0.01
C       ALPHA--ORIENTATION ANGLE OF SAMPLER
C
C
C   CARD 11:---11+IM/8        Z(I)   FORMAT(8F10.4)
C       AXIAL COORDINATES OF GRID POINTS. FOR NDIM=0,1 Z(I) IS CALCULATED
C       BY 'FLOWFI'. FOR NDIM=2 INPUT BY USER. THERE WILL BE
C       IM/8 DATA CARDS.
C
C   CARDS 11+IM/8--11+IM/8+JM/8=N(SAY)   R(I)   FORMAT(8F10.4)
C       RADIAL COORDINATES OF GRID POINTS,FOR NDIM=0,1 R(I) IS CALCULATED
C       BY 'FLOWFI'. FOR NDIM=2 INPUT BY USER. THERE WILL BE
C       JM/8 DATA CARDS.
C
C   CARD N--N+IM*JM/8 =M(SAY)   UZ(I,J)    FORMAT(8F10.4)  OMIT IF NDIM=2
C       UZ--AXIAL COMPONENT OF VELOCITY. FOR NDIM=0 OR 1 CALCULATED BY
C       'FLOWFI'
C
C   CARDSM--M+IM*JM/8=MM(SAY)   UR(I,J)   FORMAT(8F10.4)   OMIT IF NDIM=2
C       UR-- RADIAL COMPONENT OF VELOCITY. FOR NDIM=0,1 CALCULATED BY FLOWFI
C
C   CARD (MM+1)   LIM,DTI,ZI,RI,DR         FORMAT(I5,4F15.6)
C       LIM--# OF TIMES TREJECTORY CALCULATION HAS TO BE DONE
C       DTI--TIME INCREAMENT FOR EACH STEP (0.1)
C       ZI--INITIAL AXIAL POSITION OF THE PARTICLE.
C       RI--INITIAL RADIAL POSITION OF PARTICLE=SQRT(U)
C       DR-- RADIAL INCREAMENT (0.2)
C
C   CARD(MM+2) UINF,RE FORMAT(2F15.6)
C       UINF--FREE STREAM VELOCITY USER INPUT
C       RE-- REYNOLD'S NUMBER BASED ON TUBE WIDTH
C
C   CARD(MM+3): ZLIP   FORMAT(F15.6)
C       ZLIP--LIP DEPTH OF SAMPLER.
C
C   CARD(MM+4): UP,QC,PL   FORMAT(3F10.4) OMIT IF IE=0
C       UP--ELECTROSTATIC CHARGE ON THE PARTICLE
C       QC--ELECTROSTATIC CHARGE ON THE SAMPLER.
C       PL--LINEAR DIMENSION OF THE SAMPLER.
C
C   CARD(MM+5): IST    FORMAT(I4)
C       # OF SIZE INTERVALS IN THE SIZE DISTRIBUTION  <10
C
C   CARDS(MM+6)--(MM+6+IST)   P,TA,PF      FORMAT(3F15.8)
C
```

```
C         P--PARTICLE RADIUS IN MICRONS
C         TA--RELAXATION TIME OF THE PARTICLE IN SEC
C         PF--MEASURED FRACTION OF PARTICLES
C         THERE WILL BE 'IST' DATA CARDS.
C*************************************************************************
C*************************************************************************
C
C
C
C
C
C
C
      PROGRAM INER
      DIMENSION ST(10),TA(10),PF(10),P(10)
      COMMON/AAA/STK,FR
      COMMON/ABA/R(61),Z(101)
      COMMON/ABB/UZ(101,61),UR(101,61)
      COMMON/ABC/IM,JM,JGR,JGRO,IBW,IBWO
      COMMON/ACB/T,IC,INT,ILP,ZLIP
      COMMON/BAT/NDIM,ISINK,F,ALPHA,U,IDIM
      COMMON/SAR/IS,IE,QP,QC,IW,PL,ESP,ITS
      ITS=0
      READ(5,1) INT,
      READ(5,1) IE,IW,IS
      READ(5,2) U,W,ANG
      READ(5,1) NDIM
      IF(NDIM.EQ.2) READ(5,1)IDIM
      READ(5,1)JGR,JGRO
      READ(5,1) IBW,IBWO
      READ(5,1)IM,JM
      WRITE(6,19)
   19 FORMAT(1H1,30X,'SAMPLING BIAS DETERMINATION'////,
     1       1H,'TRAJECTORY CALCULATION')
    1 FORMAT(3I4)
    2 FORMAT(8F10.4)
    3 FORMAT( )
      WRITE(6,22) U,W,ANG
   22 FORMAT(//5X,'VEL.RATIO=',F10.6,/5X,'WALL THICKNESS=',
     1F10.6,/5X,'TAPERING ANG=',F10.6,'DEG')
      IF(NDIM.EQ.2) READ(5,1) ISINK
      IF(NDIM.EQ.2) READ(5,2) F,ALPHA
      PI=3.14159E2
      IF(NDIM.NE.2) ALPHA=PI/2.
      IF(NDIM.EQ.2) GO TO 29
   24 READ(5,2)(Z(I),I=1,IM)
      READ(5,2)(R(I),I=1,JM)
      IF(NDIM.EQ.2) GO TO 28
      READ(5,2)((UZ(I,J),I=1,IM),J=1,JM)
      READ(5,2)((UR(I,J),I=1,IM),J=1,JM)
   28 ZMAX=0.
      ZRU=-1.
      READ(5,4)LIM,DTI,ZI,RI,DR
      RI=U
      IF(NDIM.NE.0) RI=SQRT(U)
      READ(5,5) UINF,RE
      READ(5,2) ZLIP
      FILP=(ZLIP-Z(1))/(Z(2)-Z(1))
      ILP=FILP+1.
      ZMAX=ZLIP
      IF(IF.EQ.1) READ(5,2) QP,QC,PL
```

```
 4 FORMAT(I5,4F15.6)
 5 FORMAT(2F15.6)
   READ(5,1) IST
   CALL BIAS(IST,ST,P,TA,PF,ICA,UINF,RE)
   ALPH=ALPHA/PI*180.
   WRITE(6,30)
30 FORMAT(//25X,'FLOW FIELD BOUNDARY'///)
   IF(NDIM.EQ.0) WRITE(6,31)
   IF(NDIM.EQ.1) WRITE(6,32)
   IF(NDIM.EQ.2) WRITE(6,33) ALPH,ISINK,F,IDIM
31 FORMAT(1H,10X,'FLOW FIELD FOR TWO PARALLEL PLATES',///,10X,
  1'TWO DIMENSIONAL CASE'//)
32 FORMAT(1H,10X,'FLOW FIELD FOR CIRCULAR TUBE',///,10X,
  1'AXIALLY SYMMETRIC CASE'//)
33 FORMAT(1H,9X,'FLOW FIELD WITH A LINE SINK',///,10X,
  1'ORIENTATION ANGLE ALPHA=',F10.6,'DEGREES TO THE FLOW',///,
  210X,'SINK DISTRIBTUION PARAMETER ISINK=',I4,///,
  310X,'SINK DISTRIBUTED OVER',F10.6,'OF THE PROBE DIAMETER',///,
  410X,'IDIM=',I4,///,)
34 FORMAT(//,1H,25X,'WALL PARAMETER IW=',I4//)
   WRITE(6,39) ZLIP
39 FORMAT(    1H,10X,'ZLIP=',F15.6//)
   WRITE(6,35) IS,IE,ITS
35 FORMAT(1H,9X,'SEDIMENTATION PARAMETER=',I4,///,
  110X,'ELECTROSTATIC FIELD PARAMETER=',I4,///,
  110X,'SEDIMENTATION TRAJECTORY PARA ITS=',I4,//)
   WRITE(6,36)
36 FORMAT(1H,'RADIUS',3X,'FRACTION',/,1H,'MICRON')
   DO 37 I=1,IST
37 WRITE(6,2) P(I),PF(I)
   LIMIT=20
   CAT=1.05
   EPPS=1E-5
   EPP=10.*EPPS
   ZIO=ZI
   DT1=DTI
   DT2=5.*DTI
   DT3=DTI/10.
   DT3=DTI/5.
   DT4=DT3/2.
   IF(LIMIT.LT.1)STOP
   IWP=1
   IF(NDIM.EQ.2.AND.ALPHA.LT.1.57) IWP=2
   FWP=IWP
   YUP=0.
   YLO=0.
   DO 24 II=1,IST
   IF(II.NE.1) DR=0.1
   STK=ST(II)
   TAU=TA(II)
   PP=P(II)
   ESP=3.*DP*DC/4./PI/PP**3/UINF**2*10.**12
   G=981.
   IF(IS.LT.1) GO TO 6
   VS=TAU*G
   GO TO 17
 6 VS=0.
17 RT=RE*.1467/UINF
   FR=UINF**2/RT/G
   WR=PP*10**(-4)/RT
```

```
      VS=VS/UINF
      ZI=ZIO
      WRITE(6,2)ZI,PP,RE,UINF,STK,FR,RT
      DO 38 II=1,IWP
      IW=1
      IF(II.EQ.2) IW=-1
      FIW=IW
      WRITE(6,34) IW
      K=0
      IF(IW.EQ.1.AND.II.EQ.1) RO=RI
      IF(IW.EQ.1.AND.II.NE.1) RO=YUP
      IF(II.EQ.2.AND.II.EQ.1) RO=YUP-1.5
      IF(II.EQ.2.AND.II.NE.1) RO=YLO
      RCA=0.
      RES=0.
      FI1=0.
      DO 20 I=1,LIM
      IC=0
      KODE=0
      J=0
      IHAL=0
      IDIV=0
      DT=DT3
      DT=DT2
      DT=DT1
      IF(STK.LT..05) DT=DT3
      IF(K.EQ.2) FI1=1.
      IF(K.EQ.1) FI1=-1.
      IF(K.EQ.4) FI1=ABS(FI1)/2.
      IF(K.EQ.5) FI1=-ABS(FI1)/2.
      RO=RO+DR*FI1*FIW
      IF(NDIM.NE.2.AND.RO.LT.0.) STOP
      ROO=RO
      T=0.
      ICAL=0
      ZI=ZIO
      ZII=ZI*SIN(ALPHA)-RO*COS(ALPHA)
      RO=ZI*COS(ALPHA)+RO*SIN(ALPHA)
      ZI=ZII
      CALL OUTP(ZI,RO,KODE)
      IC=1
      CALL INTERP(RO,ZI,UX,UY,DUX,DUY)
      VZ=UX
      VR=UY
   14 CALL FCT(RO,ZI,DVZ,DVR,UX,UY,VZ,VR,DUX,DUY)
      DUXO=DUX
      DUYO=DUY
      DVZO=DVZ
      DVRO=DVR
      IF(IDIV.EQ.1)DT=DT1
      IF(IDIV.EQ.2)DT=DT3
      IF(IDIV.EQ.3)DT=DT4
      IF(STK.LT..50.AND.ZI.GE.ZCL) DT=DT4
    6 M=1
C     PREDICT NEW POSITION
    9 IF(STK.LT..1) GO TO 42
      VZP=VZ+DVZ*DT
      VRP=VR+DVR*DT
      GO TO 43
   42 VZP=UX/(1.+STK*DUX)
```

```
      VRP=UY/(1.+STK*DUY)
   43 ZP=ZI+VZP*DT
      RP=RO+VRP*DT
      RAT=RP/RU
      IF(ICAT.EQ.1) GO TO 10
      IF(RAT.GT.CAT)DT=DT/2.
      IF(DT.LE.1E-3) ICAT=1
      IF(RAT.GT.CAT)GO TO 9
C     CORRECT THE PREDICTED POSITION
   10 CALL FC1(RP,ZP,CVZ,CVR,UXC,UYC,VZP,VRP,CUX,CUY)
      VZC=VZ+(DVZ+CVZ)/2.*DT
      VRC=VR+(DVR+CVR)/2.*DT
      IF(STK.LT..1)VZC=UX/2./(1.+STK*DUX)+UXC/2./(1.+STK*CUX)
      IF(STK.LT..1)VRC=UY/2./(1.+STK*DUY)+UYC/2./(1.+STK*CUY)
      ZC=ZI+VZC*DT
      RC=RO+VRC*DT
C     ITERATE FOR CONVERGENCE
      IF(M.EQ.1) GO TO 11
      IF(ABS(ZC1-ZC).LE.EPS.AND.ABS(RC1-RC).LE.EPS)GO TO 12
   11 M=M+1
      IF(M.GT.LIMIT) IDIV=IDIV+1
      IF(IHAL.NE.0.AND.M.LE.LIMIT) GO TO 7
      IF(IDIV.GT.5) GO TO 13
      IF(M.GT.LIMIT) GO TO 14
    7 ZP=ZC
      RP=RC
      VZP=VZC
      VRP=VRC
      RC1=RC
      ZC1=ZC
      GO TO 10
   13 DT=DT/2.
      IHAL=IHAL+1
      IF(IHAL.GE.5)WRITE(6,5)ZI,RO,ZC,ZP,RP,RC,CVZ,CVR
      IF(IHAL.GE.5)STOP
      DVZ=DVZO
      DVR=DVRO
      DUX=DUXO
      DUY=DUYO
      GO TO 8
   12 KODE=0
      T=T+DT
      CALL OUTP(ZC,RC,KODE)
      IF(KODE.EQ.1) WRITE(6,15)
   15 FORMAT(//5X,'CAPTURED')
      IF(KODE.NE.1) GO TO 21
      RCA=RCO
      IF(K.EQ.0) K=2
      IF(K.EQ.1.OR.K.EQ.3) K=4
      GO TO 23
   21 CONTINUE
      ZI=ZC
      RO=RC
      VZ=VZC
      VR=VRC
      IF(NDIM.NE.2.AND.RO.LT.0.) STOP
      IHAL=0
      IF(ALPHA.EQ.0.AND.RO.GE.1.) GO TO 41
      IF(ZI.LT.ZMAX) GO TO 14
   41 WRITE(6,27) T,ZI,RO
```

```
      WRITE(6,18)
   18 FORMAT(//5X,'ESCAPED')
   27 FORMAT(1H,3(F15.6,10X))
      IF(K.EQ.0) K=1
      IF(K.EQ.2.OR.K.EQ.4) K=3
      RES=R00
   23 R0=R00
      IF(ABS(RCA-RES).LT.EPP ) GO TO 25
   20 CONTINUE
   25 IF(IIN.EQ.1) YUP=RCA/2.+RES/2.
      IF(IIN.EQ.2) YLO=RCA/2.+RES/2.
      WRITE(6,40) CAT,ICAT
   40 FORMAT(//,1H,10X,'MAX. POSITION RATIO=',F15.6,//,10X,
     1'VALUE OF ICAT IS=',I4,//)
   38 CONTINUE
      EFF=((YUP-YLO)/FWP)**2/U
      IF(NDIM.NE.1.AND.IDIM.EQ.0) EFF=((YUP-YLO)/FWP)/U
      EFF=EFF+VS*COS(ALPHA)/U
      WRITE(6,26) U,EFF,STK,YUP,YLO
   26 FORMAT(//5X,'VAL. RATIO=',F10.6,/5X,'CONC. RATIO=',F10.6,
     1/5X,'STOKES NUMBER=',F10.6,/5X,'CRITICAL ORD.=',F10.6,5X,F10.6)
      PF(II)=PF(II)/EFF
   24 CONTINUE
      CALL BIAS(IST,ST,P,TA,FF,ICA,LINE,KE)
      STOP
      END
$FOR,I FCT
      SUBROUTINE FCT(RO,ZO,DVZ,DVR,UX,UY,VX,VY,DUX,DUY)
      COMMON/AAA/STK,FR
      COMMON/BAT/NDIM,ISINK,F,ALPHA,U,IDIM
      COMMON/SAR/IS,IE,OP,OL,IW,PI,ESP,ITS
      FIW=IW
      CALL INTERP(RO,ZO,UX,UY,DUX,DUY)
      IF(STK.LT..1) GO TO 2
      DVZ=(UX-VX)/STK
      DVR=(UY-VY)/STK
      IF(IS.EQ.0) GO TO 1
      DVZ=DVZ+COS(ALPHA)/FR*FIW
      DVR=DVR-SIN(ALPHA)/FR*FIW
    1 IF(IE.EQ.0) GO TO 2
      FUN=SQRT((RO-1.)**2+ZO**2)
      S=(RO-1.)/FUN
      C=ZO/FUN
      FUN=SQRT((RO-1.)**2+(PL-ZO)**2)
      SS=(RO-1.)/FUN
      CC=-(PL-ZO)/FUN
      FX=ESP/(RO-1.)*(S-SS)
      FY=ESP/(RO-1.)*(CC-C)
      DVZ=DVZ+FX
      DVR=DVR+FY
    2 RETURN
      END
$FOR,I INTERP
      SUBROUTINE INTERP(RO,ZO,UX,UY,DUX,DUY)
      COMMON/AAA/R(61),Z(101)
      COMMON/BAT/NDIM,ISINK,F,ALPHA,U,IDIM
      COMMON/ABB/UZ(101,61),UR(101,61)
      COMMON/ABC/IM,JM,JGR,JGRO,IBW,IHWO
      IF(NDIM.EQ.2) CALL SINVEL(RO,ZO,UX,UY,DUX,DUY)
      IF(NDIM.EQ.2) RETURN
```

```
      DO1 I=1,IM
      IF(ZO.GE.Z(I)  .AND.ZO.LT.Z(I+1))GO TO 2
    1 CONTINUE
    2 IF(I.EQ.IHW) I=I-1
      DO 3 J=1,JM
      IF(RO.GE.R(J).AND.RO.LT.R(J+1))GO TO 4
    3 CONTINUE
    4 CONTINUE
      UXL=(UZ(I+1,J)-UZ(I,J))/(Z(I+1)-Z(I))* (ZO-Z(I))
      UYL=(UR(I+1,J)-UR(I,J))/(Z(I+1)-Z(I))* (ZO-Z(I))
      UXU=(UZ(I+1,J+1)-UZ(I,J+1))/(Z(I+1)-Z(I))*(ZO-Z(I))
      UYU=(UR(I+1,J+1)-UR(I,J+1))/(Z(I+1)-Z(I))*(ZO-Z(I))
      UYL=UYL+UR(I,J)
      UXU=UXU+UZ(I,J+1)
      UYU=UYL+UR(I,J+1)
      UXL=UXL+UZ(I,J)
      UX=(UXU-UXL)/(R(J+1)-R(J))*(RO-R(J))
      UY=(UYU-UYL)/(R(J+1)-R(J))*(RO-R(J))
      DUY=(UYU-UYL)/(R(J+1)-R(J))
      DUX=UZ(I+1,J)-UZ(I,J)+(UZ(I+1,J+1)-UZ(I+1,J)-UZ(I,J+1)+UZ(I,J))
     1*(RO-R(J))/(R(J+1)-R(J))
      DUX=DUX/(Z(I+1)-Z(I))
      UX=UX+UXL
      UY=UY+UYL
      RETURN
      END
#FOR,I SINVEL
      SUBROUTINE SINVEL(RO,ZO,UX,UY,DUX,DUY)
      COMMON/MAT/NDIM,ISINK,F,ALPHA,U,IDIM
      COMMON/SAM/IS,IE,OP,OC,IW,PL,ESP,ITS
      COMMON/ACM/T,IC,INT,ILP,ZLIP
      FUN(X,Y)=SQRT(X**2+Y**2)
      THIL(X,Y)=(X**2-Y**2)/(X**2+Y**2)
      ARA(X,Y,Z)=ALOG((X**2+Y**2)/(Z**2+Y**2))
      ACM(X,Y)=1./(X**2+Y**2)
      REAL M
      C=COS(ALPHA)
      S=SIN(ALPHA)
      M=(U-S)/4./F
      IF(ISINK.EQ.1) M=(S-U)/2./F**2
      PI=3.141596
      THETA=PI-2.*ATAN(ZLIP)
      M=M*PI/THETA
      X=RO+F
      Y=-ZO
      XF=X-F
      XFF=XF-F
      IF(IDIM.EQ.0) GO TO 1
      IF(ISINK.EQ.0) GO TO 13
      IF(ABS(Y).LE..01) GO TO 10
      A=FUN(X,Y)
      B=FUN(XF,Y)
      D=FUN(XFF,Y)
      H=B+XF
      E=XFF+D
      G=X+A
      UUZ=-X/2./A+XF/B-XFF/2./D+ALOG(H/G)+ALOG(F/E)+
     1Y**2/2.*(2./H/B-1./G/A-1./E/D)
      UUR=X**2/2./Y/A-XF**2/Y/B+XFF**2/2./Y/D-Y/2.*(2./B-1./A-1./D)
      UUR=UUR+A/2./Y-B/Y+D/2./Y
```

```
      DUY=2./H-1./A-1./D
      DUX=-X**2/A+2.*XF**2/H-XFF**2/D
      DUX=DUX*M/Y**2
      DUY=DUY*M
      UUZ=UUZ*M
      UUR=UUR*M
      GO TO 11
   10 IF(X.GE.0..AND.X.LE.(2.*F)) GO TO 12
      UUR=0.
      UUZ=M*ALOG(XF**2/X/XFF)
      GO TO 11
   12 UUZ=0.
      UUR=0.
      GO TO 11
   15 A=FUN(X,Y)
      H=FUN(XFF,Y)
      IF(A.LE..01) A=.01
      IF(H.LE..01) H=.01
      UUZ=M*(1./A-1./H)
      UUR=-M*(X/A/Y-(XFF)/B/Y)
      DUY=M*(-X/A**3+XFF/H**3)
      DUX=-M*(-X/A**3-X/A/Y**2+XFF/B**3+XFF/B/Y**2)
      GO TO 11
    1 M=4.*(S-U)/2./F**2/PI
      IF(ISING.EQ.0) M=(U-S)/PI/F
      M=F*PI/THETA
      IF(ISING.EQ.0) GO TO 2
      A=THIL(X,Y)
      B=THIL(XF,Y)
      D=THIL(XFF,Y)
      E=ARA(X,Y,XF)
      G=ARA(XF,Y,XFF)
      H=ACH(X,Y)
      P=ACH(XF,Y)
      Q=ACH(XFF,Y)
      AA=ATAN(Y/X)
      AR=ATAN(Y/XF)
      AC=ATAN(Y/XFF)
      UUZ=X/2.*A-Y*AA-XF*B+2.*Y*AR+XFF/2.*D-Y*AC+X/2.*E-XFF/2.*G+Y**2
     1*X*(H-P)-Y**2*XFF*(P-Q)
      UUR=Y/2.*A-X*AA-Y*B+2.*XF*AR+Y/2.*D-XFF*AC-Y/2.*E+Y/2.*G-Y*X*
     1(X*H-XF*P)+Y*XFF*(XF*P-XFF*Q)
      DUY=E/2.-G/2.
      DUX=-E/2.+G/2.
      IF(ABS(RO).LE.F) UUZ=0.
      IF(ABS(RO).LE.F) UUR=-F**2/2./Y
      UUZ=UUZ*M
      UUR=UUR*M
      DUY=DUY*M
      DUX=DUX*M
      GO TO 11
    2 A=ARA(X,Y,XFF)
      B=ATAN(Y/X)
      D=ATAN(Y/XFF)
      G=ACH(X,Y)
      H=ACH(XFF,Y)
      UUZ=-M*A
      UUR=M*(B-D)
      DUY=-M*(X*G-XFF*H)
      DUX=F*(X*G-XFF*H)
```

```
   11 CONTINUE
      UX=-UUR*S
      UY=UUZ*C
      RETURN
      END
*FOR,I BIAS
      SUBROUTINE BIAS(IST,ST,P,TA,PF,ICA,UINF,RE)
      DIMENSION ST(IST),P(IST),TA(IST),PF(IST)
      TPF=0.
      IF(ICA.EQ.1) GO TO 1
      DO 2 I=1,IST
      READ(5,3) P(I),TA(I),PF(I)
      ST(I) =UINF**2*TA(I)/RE/.1467
    2 CONTINUE
      ST(1)=.008
      ST(2)=.040
      ST(3)=.105
      ST(4)=.464
      ST(5)=1.744
      ST(6)=5.224
    3 FORMAT(3F15.8)
      ICA=1
      RETURN
    1 DO 4 I=1,IST
      TPF=TPF+PF(I)
    4 CONTINUE
      DO 5 I=1,IST
      PF(I)=PF(I)/TPF
    5 CONTINUE
      WRITE(6,36)
   36 FORMAT(1H,35X,'ACTUAL DISTRIBUTION',//,
     15X,'RADIUS',11X,'FRACTION',/5X,'MICRON'/)
      DO 6 I=1,IST
      WRITE(6,3) P(I),PF(I)
    6 CONTINUE
      RETURN
      END
*FOR,I OUTP
      SUBROUTINE OUTP(ZP,RP,KODE)
      COMMON/ABA/R(61),Z(101)
      COMMON/ABC/IM,JM,JGR,JGRO,IBW,IRWO
      COMMON/ACB/T,IC,K,ILP,ZLIP
      COMMON/BAT/NDIM,ISINK,F,ALPHA,U,IDIM
      IF(IC.EQ.0) WRITE(6,7) T,ZP,RP
      PP=ABS(RP)
      N=N+1
      IF(N.EQ.K)WRITE(6,7) T,ZP,RP
      IF(N.EQ.K) N=0
    7 FORMAT(1H,3(F15.6,10X))
      IF(NDIM.EQ.2) GO TO 8
      DO 1 I=1,IM
      IF(ZP.GE.Z(I).AND.ZP.LT.Z(I+1)) GO TO 2
    1 CONTINUE
    2 IF(I-ILP)3,4,4
    8 IF(ZP-ZLIP)3,4,4
    4 DO5 J=1,JM
      IF(PP.GE.R(J).AND.PP.LT.R(J+1)) GO TO 6
    5 CONTINUE
    6 IF(J.GE.JGR.AND.J.LE.JGRO)KODE=2
      IF(J.LT.JGR) KODE=1
```

```
      IF(KUCE.NE.0)WRITE(6,7)T,ZP,RP
    3 RETURN
      END
**
```

Appendix E. Computer Program System - User Manual

The computer program system consists of two separate programs. Program 'FLOWFI' solves for the flow field and program 'TRAJEC' computes the particle trajectories in the specified flow region. Since the model uses the stream function equation only when the sampling head faces the stream, program 'FLOWFI' has to be run only with this option. For angular orientations the flow field is approximated by a line sink/source in a uniform stream and the flow field is incorporated in the 'TRAJEC' program. User instructions for both programs follow.

The fluid flow model covers various inlet geometries:

- Parallel plate inlet facing the stream
- Circular tube inlet facing the stream
- Slit inlet with arbitrary orientation to the stream

The user can choose any of the above options with proper value of an integer parameter 'NDIM'.

NDIM = 0 Parallel plate inlet

NDIM = 1 Circular tube

NDIM = 2 Arbitrary orientation - slit

For options 0 or 1, it is necessary to obtain the flow field data from program 'FLOWFI' to do the trajectory calculations. However, for option 2 the flow field is incorporated in the program 'TRAJEC' and the use of 'FLOWFI' is not required. Various inlet geometries and physical conditions are simulated by proper selection of parameters U, W, ANG, NDIM, ZLIP, ALPHA. Figure 37 shows the physical meaning of these parameters.

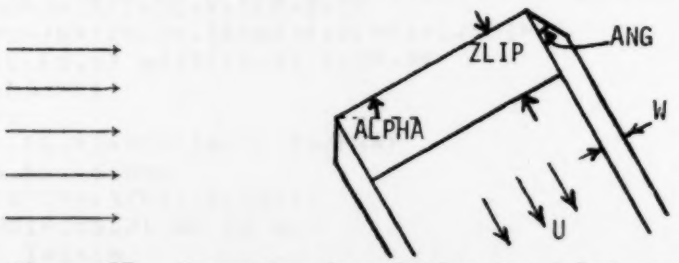

Figure 37. Parameter description for inlet geometry and orientation.

The 'FLOWFI' program input data and their description follows. The recommended values for some non-physical variables are also given.

```
C                          'FLOWFI' DATA INPUT
C                           ******************
C
C   CARD 1:   NDIM,IPUNCH    FORMAT( )
C
C             NDIM.....FLOW FIELD OPTION PARAMETER
C                      =0 TWO DIMENSIONAL CASE
C                      =1 AXISYMMETRIC CASE
C                      =2 LINE SINK
C
C             IPUNCH....CONTROL PARAMETER FOR GETTING PUNCHED OUTPUT OF
C                      PSI,UZ,UR,Z,R,ERC
C                      =0 NO PUNCHED OUTPUT DESIRED.
C                      =1 PUNCHED OUTPUT DESIRED.
C
C   CARD 2:   ISINK,ALPHA,F   FORMAT(I5,2F10.4)    OMIT IF NDIM.NE.2
C
C             ISINK.....SINK DISTRIBUTION PARAMETER
C                      =0 LINE SINK OF UNIFORM STRENGTH.
C                      =1 LINE SINK OF TRIANGULAR STRENGTH DISTRIBUTION.
C
C             ALPHA.....ANGULAR ORIENTATION OF THE FACE PLANE OF SAMPLER HEAD.
C                      =90 DEGREES WHEN SAMPLER FACES THE STREAM
C                      =0  DEGREES WHEN STREAM IS TANGENTIAL TO THE HEAD.
C             F.........FRACTION OF THE PROBE DIAMETER OVER WHICH
C                      THE SINK IS ASSUMED TO BE DISTRIBUTED.
C                      RECOMMENDED VALUE IS 0.01.
C
C   CARD 3:   ITERMA,RELAX   FORMAT(I5,F10.4)
C
C             ITERMA....MAXIMUM ALLOWABLE ITERATIONS FOR FLOW FIELD
C                      TO CONVERGE
C
C             RELAX.....RELAXATION PARAMETER USED IN THE PROCEDURE OF
C                      SUCCESSIVE OVER RELAXATION(SOR)
C                      VALUES BETWEEN 1.0 AND 2.0.
C
C   CARD 4:   ZM,ZMA,RO,U,W,ANG      FORMAT(6F10.4)
C
C             ZM........UPSTREAM FLOW FIELD BOUNDARY.LE.-5.
C
C             ZMA.......DOWNSTREAM FLOW FIELD BOUNDARY
C
C             RO........FLOW FIELD BOUNDARY IN THE RADIAL OR TRANSVERSE
C                      DIRECTION.  RO.GE.5.
C
C             U.........ANISOKINETIC VELOCITY RATIO.
C
C             W.........SAMPLER WALL THICKNESS
C                      WHEN NDIM.EQ.2 W=0.
C
C             ANG.......TAPERING ANGLE OF SAMPLER WALL
C                      =0 IF W=0. OR NDIM.EQ.2
C
C   CARD 5:   IGZ,JGR     FORMAT(2I5)
C
C             IGZ.......GRID POINTS PER UNIT LINGTH IN AXIAL DIRECTION
C                      = 1./GRID SIZE +1        Recommended value 6 or greater
C
C             JGR.......GRID POINTS PER UNIT LINGTH IN RADIAL DIRECTION
C                      =1./RADIAL GRID SIZE +1 Recommended value 6 or greater
```

```
C
C    CARD 6:    II        FORMAT( )
C
C               II........NUMBER OF STREAM FUNCTION CONTOURS NEEDED.LE.10
C
C    CARD 7:    STR       FORMAT( )
C
C               STR.......STREAM FUNCTION VALUES FOR THE CONTOUR.THIS ARRAY
C                         WILL CONTAIN 'II' VALUES
C
C    ***************************************************************************
C
C
**                                                                              **
```

Program 'TRAJEC' solves for the limiting particle trajectory and calculates the true distribution of particle size from the measured distribution.

Program 'TRAJEC' starts the trajectory calculation at position ZI,RI input by the user. ZI is the upstream distance at which the particle is moving with the stream and is usually at least 5 radii from the sampling head. RI is the radial position to start the process to find RC, the radial position of limiting trajectory at ZI. If the trajectory of the particle starting at position RI enters the probe, then the next trajectory is started from a position RI = RI + DR, where DR is a preselected radial increment. If the trajectory from RI escapes the sampler, then the new RI is given by RI-DR. This process continues until the trajectories from successive radial positions alternate (i.e., one gets captured and another escapes). This process is repeated with successive halving of DR until the radial positions RES (at which the particle escapes) and RCA (at which the particle gets captured) are within a preselected tolerance EPP.

To calculate the trajectory accurately, it is necessary that the time or space step is not very large. This is accomplished by taking the ratio RAT of predicted radial position RP to initial radial position RD. If the ratio RAT is greater than a prespecified value CAT then the time/space step is halved and recalculation starts. The value of CAT used in the program is 1.05. Even with a step size that satisfies the ratio test, if the iterative convergence fails within LIMIT iteration than the step size is halved and the calculation procedure is restarted. The origin of the coordinate system lies at the center of the sampling head. The Z axis is along the centerline and R axis along the face plane. The efficiency calculation is performed for IST number of particles.

Subroutine BIAS calculates the actual size distribution from the measured size distribution. It uses the efficiency calculated by the subroutine INER. The Stokes number of a given particle K is also calculated by BIAS. The measured fraction PF, particle size P, relaxation time TAU are input to the subroutine BIAS. The maximum number of particle intervals that can be used is 10.

The input data and their description follows. The recommended values for non-physical variables are also given.

```
                    TRAJEC DATA INPUT

))))))
```

```
CARD 11:---11+IM/R        Z(I)   FORMAT(8F10.4)
     AXIAL COORDINATES OF GRID POINTS. FOR NDIM=0,1 Z(I) IS CALCULATED
     BY 'FLOWFI'. FOR NDIM=2 INPUT BY USER. THERE WILL BE
     IM/R DATA CARDS.

CARDS 11+IM/R---11+IM/R+JM/R=N(SAY)   R(I)   FORMAT(8F10.4)
     RADIAL COORDINATES OF GRID POINTS.FOR NDIM=0,1 R(I) IS CALCULATED
     BY 'FLOWFI'. FOR NDIM=2 INPUT BY USER. THERE WILL BE
     JM/R DATA CARDS.

CARD N---N+IM*JM/R =M(SAY)   UZ(I,J)    FORMAT(8F10.4)  OMIT IF NDIM=2
     UZ--AXIAL COMPONENT OF VELOCITY. FOR NDIM=0 OR 1 CALCULATED BY
     'FLOWFI'

CARDSM---M+IM*JM/R=MM(SAY)   UR(I,J)   FORMAT(8F10.4)   OMIT IF NDIM=2
     UR-- RADIAL COMPONENT OF VELOCITY. FOR NDIM=0,1 CALCULATED BY FLOWFI

CARD (MM+1)   LIM,DTI,ZI,RI,DR          FORMAT(I5,4F15.6)
     LIM--# OF TIMES TRAJECTORY CALCULATION HAS TO BE DONE
     DTI--TIME INCREAMENT FOR EACH STEP (0.1)
     ZI--INITIAL AXIAL POSITION OF THE PARTICLE.
     RI--INITIAL RADIAL PLSITION OF PARTICLE=SQRT(0)
     DR-- RADIAL INCREAMENT (0.2)

CARD(MM+2) UINF,RE FORMAT(2F15.6)
     UINF--FREE STREAM VELOCITY USER INPUT
     RE-- REYNOLD'S NUMBER BASED ON TUBE WIDTH

CARD(MM+3): ZLIP   FORMAT(F15.6)
     ZLIP--LIP DEPTH OF SAMPLER.

CARD(MM+4): QP,QC,PL   FORMAT(3F10.4) OMIT IF IE=0
     'QP--ELECTROSTATIC CHARGE ON THE PARTICLE
      QC--ELECTROSTATIC CHARGE ON THE SAMPLER.
      PL--LINEAR DIMENSION OF THE SAMPLER.

CARD(MM+5): IST    FORMAT(I4)
     # OF SIZE INTERVALS IN THE SIZE DISTRIBUTION  <10

CARDS(MM+6)--(MM+6+IST)   P,TA,PF      FORMAT(3F15.8)

     P--PARTICLE RADIUS IN MICRONS
     TA--RELAXATION TIME OF THE PARTICLE IN SEC
     PF--MEASURED FRACTION OF PARTILLES
     THERE WILL BE 'IST' DATA CARDS.
))))))
```

For NDIM options of 0 or 1, the data cards 3 through 'MM' are punched out in the same order by program 'FLOWFI'. However, for NDIM=2, the user has to calculate the axial and radial coordinates as shown below.

The flow region of interest is given by upstream boundary (ZM), downstream boundary (ZMA) and radial boundary (RO). Assuming the face of the sampler is located at the origin, then

$$IM = \frac{-ZM + ZMA}{\Delta Z} + 1$$

$$IBW = \frac{-ZM}{\Delta Z} + 1$$

$$JM = \frac{RO}{\Delta R} + 1$$

$$JGR = \frac{1}{\Delta R} + 1$$

where ΔZ, ΔR are the grid sizes in axial and radial direction. Coordinates of the grid points are

$$Z(I) = ZM + \Delta Z * (J-1) \qquad \text{for } I = 1, IM$$

$$R(J) = \Delta R * (J-1) \qquad \text{for } J = 1, JM$$

For typical values of ZM=-5, ZMA=5, RO=5 and $\Delta Z=\Delta R=0.2$, IM=51, IBW=26, JM=26 and JGR=6.

The axial and radial velocity data cards are omitted for option NDIM=2.

EXAMPLE PROBLEM

LET US ASSUME A CIRCULAR TUBE OF 0.93 CM RADIUS AND A WALL THICKNESS OF 0.09 CM IS SAMPLING FROM A STREAM AT A VELOCITY OF 300CM/SEC. THE WIND SPEED IS 200CM/SEC. THE SAMPLER FACES THE STREAM. THE MEASURED PARTICLE SIZE DISTRIBUTION IS GIVEN BELOW.

RADIUS(MICRONS)	FRACTION
3.5	0.1
7.5	0.28
12.5	0.23
20.5	0.32
51.0	0.06

CALCULATE THE TRUE PARTICLE SIZE DISTRIBUTION.

FOR THE ABOVE PROBLEM THE 'INDIM' OPTION OF 1 HAS TO BE USED.

THE VELOCITY RATIO U=300/200=1.5

THE WALL THICKNESS W= 0.09/0.92=0.1

SINCE NO CHAMFER ANGLE IS GIVEN THE TAPERING ANGLE 'ANG' =90.

FOR THE INDIM OPTION OF 1 THE RESULTS FROM PROGRAM 'FLOWFI' ARE REQUIRED TO RUN PROGRAM 'TRAJEC'. THE INPUT DATA FOR PROGRAM 'FLOWFI' IS AS FOLLOWS (REFER TO DATA INPUT DESCRIPTION FOR EXPLANATION AND FORMAT)

```
CARD 1      1     1
CARD 2      40    1.5
CARD 3     -5.    5.    5.    1.5    0.1    90.
CARD 4      6     6
CARD 5      1
CARD 6      2              (STREAM FUNCTION VALUE FOR LOCUS)
```

THE OUTPUT FROM PROGRAM 'FLOWFI' FOLLOWS.

FLOW FIELD BOUNDARY

FLOW FIELD FOR CIRCULAR TUBE

AXIALLY SYMMETRIC CASE

DISTANCE OF UPSTREAM BOUNDARY= -5.00000

DISTANCE OF DOWN STREAM BOUNDARY= 5.00000

DISTANCE OF RADIAL BOUNDARY= 5.00000

SAMPLING CONDITIONS

SAMPLING VELOCITY/FREE STREAM VELOCITY= 1.50000

SAMPLER WALL THICKNESS= .10000

TAPERING ANGLE OF SAMPLER WALL= 90.00000DEGREES

NUMBER OF GRID POINTS FOR UNIT DISTANCE Z= 6

NUMBER OF GRID POINTS FOR UNIT DISTANCE R= 6

MAXIMUM NUMBER OF POINTS IN Z DIRECTION= 51

MAXIMUM NUMBER OF POINTS IN R DIRECTION= 27

INITIAL VALUES OF STREAM FUNCTION

.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.0000	1.0000	1.0000	1.0000	1.0000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000
3.6100	3.6100	3.6100	3.6100	3.6100	3.6100	3.6100	3.6100	3.6100	3.6100	3.8719
8.4100	8.4100	8.4100	8.4100	8.4100	8.4100	8.4100	8.4100	8.4100	8.4100	8.6158
15.2100	15.2100	15.2100	15.2100	15.2100	15.2100	15.2100	15.2100	15.2100	15.2100	15.3363
24.0100	24.0100	24.0100	24.0100	24.0100	24.0100	24.0100	24.0100	24.0100	24.0100	24.0334

RELAXATION FACTOR= 1.500000

CONVERGENCE RATE

1 .8518164
2 .4670460
3 .4081648
4 .4752414
5 .1992124
6 .1737717
7 .1390101

8	.0996314
9	.0653536
10	.0402738
11	.0235050
12	.0131926
13	.0070191
14	.0040290
15	.0028434
16	.0021772
17	.0016365
18	.0015535
19	.0013509
20	.0012065
21	.0010683
22	.0009511

NUMBER OF ITERATIONS 22
MAXIMUM ERROR .0009511

FINAL STREAM FUNCTION SOLUTION

.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.0400	.0400	.0400	.0404	.0432	.0546	.0597	.0600	.0600	.0600	.0600
.1600	.1600	.1601	.1617	.1723	.2199	.2392	.2400	.2400	.2400	.2400
.3600	.3600	.3602	.3636	.3855	.5003	.5389	.5400	.5400	.5400	.5400
.6400	.6400	.6403	.6459	.6803	.9094	.9591	.9600	.9600	.9600	.9600
1.0000	1.0000	1.0005	1.0083	1.0535	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000
1.2100	1.2100	1.2107	1.2196	1.2684	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000
1.6900	1.6900	1.6910	1.7019	1.7562	1.9317	1.9607	1.9638	1.9660	1.9700	1.9744
2.2500	2.2500	2.2512	2.2637	2.3193	2.4588	2.4996	2.5058	2.5106	2.5188	2.5278
2.8900	2.8900	2.8914	2.9048	2.9589	3.0727	3.1176	3.1267	3.1341	3.1467	3.1604
3.6100	3.6100	3.6116	3.6253	3.6701	3.7705	3.8154	3.8269	3.8376	3.8538	3.8719
4.4100	4.4100	4.4117	4.4252	4.4717	4.5507	4.5936	4.6070	4.6196	4.6403	4.6626
5.2900	5.2900	5.2917	5.3047	5.3484	5.4129	5.4528	5.4674	5.4823	5.5064	5.5323
6.2500	6.2500	6.2517	6.2638	6.3006	6.3568	6.3931	6.4084	6.4252	6.4522	6.4811
7.2900	7.2900	7.2916	7.3027	7.3348	7.3822	7.4148	7.4303	7.4486	7.4779	7.5089
8.4100	8.4100	8.4115	8.4214	8.4491	8.4891	8.5181	8.5334	8.5527	8.5834	8.6158
9.6100	9.6100	9.6114	9.6201	9.6437	9.6774	9.7028	9.7176	9.7375	9.7688	9.8018
10.8900	10.8900	10.8912	10.8986	10.9188	10.9470	10.9691	10.9832	11.0031	11.0342	11.0668
12.2500	12.2500	12.2511	12.2575	12.2743	12.2978	12.3169	12.3300	12.3495	12.3795	12.4109
13.6900	13.6900	13.6909	13.6963	13.7102	13.7298	13.7460	13.7560	13.7765	13.8047	13.8341
15.2100	15.2100	15.2108	15.2153	15.2267	15.2427	15.2563	15.2671	15.2841	15.3097	15.3363
16.8100	16.8100	16.8106	16.8143	16.8235	16.8364	16.8475	16.8570	16.8721	16.8945	16.9176
18.4900	18.4900	18.4905	18.4934	18.5006	18.5107	18.5196	18.5275	18.5402	18.5588	18.5779
20.2500	20.2500	20.2504	20.2526	20.2580	20.2655	20.2722	20.2784	20.2884	20.3027	20.3174
22.0900	22.0900	22.0903	22.0918	22.0956	22.1004	22.1050	22.1093	22.1162	22.1259	22.1358
24.0100	24.0100	24.0102	24.0109	24.0128	24.0154	24.0177	24.0200	24.0235	24.0284	24.0338
26.0100	26.0100	26.0100	26.0100	26.0100	26.0100	26.0100	26.0100	26.0100	26.0100	26.0100

UZ

1.0000	1.0000	1.0004	1.0104	1.0411	1.3661	1.4937	1.4998	1.5000	1.5000	1.5000
1.0000	1.0000	1.0005	1.0105	1.0769	1.3744	1.4952	1.4999	1.5000	1.5000	1.5000
1.0000	1.0000	1.0005	1.0094	1.0696	1.3927	1.4973	1.4999	1.5000	1.5000	1.5000
1.0000	1.0000	1.0005	1.0088	1.0583	1.4174	1.4998	1.5000	1.5000	1.5000	1.5000
1.0000	1.0000	1.0005	1.0073	1.0437	1.4620	1.5018	1.5000	1.5000	1.5000	1.5000

1.0000	1.0000	1.0007	1.0064	1.0290	1.7009	1.5030	1.5001	1.5000	1.5000	.0000
1.0000	1.0000	1.0007	1.0057	1.0219	.8729	.9583	.9651	.9698	.9786	.0000
1.0000	1.0000	1.0005	1.0039	1.0100	.9219	.9612	.9671	.9717	.9796	.9878
1.0000	1.0000	1.0004	1.0024	1.0022	.9508	.9640	.9691	.9734	.9806	.9878
1.0000	1.0000	1.0003	1.0012	.9977	.9645	.9675	.9714	.9753	.9816	.9878
1.0000	1.0000	1.0002	1.0003	.9953	.9724	.9711	.9739	.9773	.9827	.9878
1.0000	1.0000	1.0001	.9998	.9942	.9777	.9746	.9765	.9793	.9837	.9878
1.0000	1.0000	1.0000	.9992	.9940	.9816	.9780	.9790	.9813	.9847	.9878
1.0000	1.0000	.9999	.9990	.9942	.9847	.9810	.9815	.9832	.9857	.9878
1.0000	1.0000	.9999	.9989	.9947	.9872	.9838	.9838	.9850	.9866	.9878
1.0000	1.0000	.9999	.9989	.9952	.9893	.9862	.9859	.9866	.9875	.9878
1.0000	1.0000	.9999	.9989	.9958	.9911	.9883	.9878	.9881	.9882	.9878
1.0000	1.0000	.9999	.9990	.9964	.9926	.9902	.9895	.9894	.9889	.9878
1.0000	1.0000	.9999	.9991	.9970	.9938	.9917	.9910	.9905	.9895	.9878
1.0000	1.0000	.9999	.9992	.9976	.9949	.9930	.9923	.9914	.9899	.9878
1.0000	1.0000	.9999	.9993	.9978	.9957	.9941	.9933	.9922	.9903	.9878
1.0000	1.0000	.9999	.9994	.9982	.9964	.9949	.9940	.9927	.9906	.9878
1.0000	1.0000	.9999	.9995	.9984	.9968	.9955	.9946	.9931	.9908	.9878
1.0000	1.0000	.9999	.9995	.9986	.9971	.9959	.9949	.9933	.9909	.9878
1.0000	1.0000	.9999	.9996	.9986	.9973	.9961	.9951	.9934	.9909	.9878
1.0000	1.0000	.9999	.9995	.9986	.9973	.9962	.9951	.9933	.9908	.9878
1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.9878

UR

.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.0000	.0000	-.0002	-.0026	-.0150	-.0312	-.0023	-.0001	-.0000	.0000	.0000
.0000	-.0000	-.0005	-.0048	-.0287	-.0669	-.0036	-.0001	-.0000	-.0000	.0000
.0000	-.0000	-.0007	-.0067	-.0400	-.1123	-.0037	-.0001	-.0000	-.0000	.0000
.0000	-.0000	-.0009	-.0081	-.0470	-.1801	-.0023	-.0001	-.0000	.0000	.0000
.0000	-.0000	-.0010	-.0089	-.0481	-.7959	.0000	.0000	.0000	.0000	.0000
.0000	-.0000	-.0012	-.0091	-.0464	-.1001	.0000	.0000	.0000	.0000	.0000
.0000	-.0000	-.0013	-.0091	-.0401	-.0479	-.0024	-.0007	-.0012	-.0017	.0000
.0000	-.0000	-.0013	-.0087	-.0350	-.0363	-.0041	-.0013	-.0022	-.0031	.0000
.0000	-.0000	-.0013	-.0080	-.0264	-.0276	-.0050	-.0018	-.0030	-.0042	.0000
.0000	-.0000	-.0013	-.0071	-.0209	-.0215	-.0053	-.0021	-.0036	-.0050	.0000
.0000	-.0000	-.0012	-.0062	-.0165	-.0165	-.0052	-.0024	-.0040	-.0055	.0000
.0000	-.0000	-.0011	-.0053	-.0130	-.0130	-.0049	-.0025	-.0043	-.0058	.0000
.0000	-.0000	-.0010	-.0045	-.0102	-.0103	-.0044	-.0026	-.0045	-.0060	.0000
.0000	-.0000	-.0008	-.0037	-.0081	-.0082	-.0039	-.0026	-.0045	-.0060	.0000
.0000	-.0000	-.0007	-.0031	-.0064	-.0065	-.0034	-.0025	-.0044	-.0058	.0000
.0000	-.0000	-.0006	-.0025	-.0050	-.0052	-.0029	-.0024	-.0042	-.0055	.0000
.0000	-.0000	-.0005	-.0020	-.0040	-.0041	-.0025	-.0022	-.0040	-.0051	.0000
.0000	-.0000	-.0004	-.0016	-.0031	-.0033	-.0021	-.0020	-.0036	-.0047	.0000
.0000	-.0000	-.0003	-.0013	-.0024	-.0026	-.0017	-.0018	-.0032	-.0041	.0000
.0000	-.0000	-.0003	-.0010	-.0019	-.0020	-.0014	-.0016	-.0028	-.0036	.0000
.0000	-.0000	-.0002	-.0008	-.0014	-.0016	-.0011	-.0014	-.0024	-.0029	.0000
.0000	-.0000	-.0002	-.0006	-.0011	-.0012	-.0009	-.0011	-.0019	-.0023	.0000
.0000	-.0000	-.0001	-.0004	-.0008	-.0008	-.0007	-.0008	-.0014	-.0017	.0000
.0000	-.0000	-.0001	-.0003	-.0005	-.0005	-.0004	-.0006	-.0009	-.0011	.0000
.0000	-.0000	-.0000	-.0001	-.0002	-.0003	-.0002	-.0003	-.0004	-.0005	.0000
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

THE PUNCHED OUTPUT RESULTING FROM THE PROGRAM 'FLOWFI' IS A PART OF THE INPUT DATA TO THE PROGRAM 'TRAJEC'. THE INPUT DATA IS AS FOLLOWS. (REFER TO DATA INPUT DESCRIPTION FOR EXPLANATION AND FORMAT)

CARD 1 50

CARD 2 0 1 0

CARD 3 PUNCHED OUTPUT FROM FLOWFI

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CARD N LAST CARD OF THE PUNCHED OUTPUT.

CARD N+1 20 0.1 -5. 1.224 0.2

CARD N+2 200. 1200.

CARD N+3 0.

CARD N+4 5

CARD N+5 3.5 3.7 E-05 0.1

CARD N+6 7.8 1.85E-04 0.28

CARD N+7 12.5 4.86 E-04 0.23

CARD N+8 20.5 2.147 E-03 0.32

CARD N+9 31.0 8.07 E-03 0.06

DUE TO THE LENGTH OF THE COMPLETE OUTPUT LISTING. THE TRAJECTORIES FOR ONE PARTICLE AND THE CORRECTED DISTRIBUTION ARE PROVIDED.

SAMPLING BIAS DETERMINATION

TRJAECTORY CALCULATION

VEL.RATIO= 1.500000
WALL THCIKNESS= .100000
TAPERING ANG= 90.000000DEG

FLOW FIELD BOUNDARY

FLOW FIELD FOR CIRCULAR TUBE

AXIALLY SYMMETRIC CASE

ZLIP= .000000

SEDIMENTATION PARAMETER= 0

ELECTROSTATIC FIELD PARAMETER= 0

SEDIMENTATION TRAJECTORY PARA ITS= 0

RADIUS(MICRONS)	FRACTION
5.5	0.1
7.5	0.28
12.5	0.23
20.5	0.32
51.0	0.06

WALL PARAMETER IW= 1

T	Z	R
.000000	-4.999998	1.224754
.980000	-4.019999	1.224754
1.980000	-3.019830	1.224403
2.979999	-2.017794	1.220267
3.979998	-1.008228	1.198568
4.489997	-.489925	1.163534
4.989997	-.016487	1.103450
5.009997	.001048	1.101434
5.009997	.001048	1.101434

ESCAPED

.000000	-4.999998	1.024754
.940000	-4.059999	1.024754
1.940000	-3.059878	1.024508
2.939999	-2.057465	1.020901
3.939998	-1.043345	.999490
4.469997	-.491433	.956265
4.889997	.000842	.844296

CAPTURED

.000000	-4.999998	1.124754
.140000	-4.859998	1.124754
1.140000	-3.859999	1.124753
2.140000	-2.859684	1.124218
3.139999	-1.856431	1.118748
4.069998	-.913105	1.093251
4.569997	-.398799	1.045445
4.929997	.013838	.909776

CAPTURED

.000000	-4.999998	1.174754
.260000	-4.739998	1.174754
1.260000	-3.739999	1.174747
2.259999	-2.739573	1.173951
3.259998	-1.735871	1.167162
4.129998	-.853696	1.140491
4.629997	-.344664	1.092050
4.949997	.001965	.991580

CAPTURED

.000000	-4.999998	1.199754
.340000	-4.659998	1.199754
1.340000	-3.659995	1.199737
2.339999	-2.659484	1.198730
3.339998	-1.655435	1.190942
4.169998	-.813938	1.163512
4.669997	-.307929	1.114391
4.989997	.007781	1.052651
4.989997	.007781	1.052651

ESCAPED

.000000	-4.999998	1.187254
.340000	-4.659998	1.187254
1.340000	-3.659995	1.187238
2.339999	-2.659475	1.186239
3.339998	-1.655329	1.178451
4.169998	-.813385	1.150797
4.669997	-.306744	1.100462
4.969997	.006864	1.021694
4.969997	.006864	1.021694

ESCAPED

.000000	-4.999998	1.181004

.380000	-4.619998	1.181004
1.380000	-3.619992	1.180984
2.379999	-2.619412	1.179879
3.379998	-1.614929	1.171548
4.189998	-.792677	1.143184
4.689997	-.286671	1.090367
4.959997	.005292	1.006424
4.959997	.005292	1.006424

ESCAPED

MAX. POSITION RATIO= 1.500000

VALUE OF ICAT IS= 0

VAL. RATIO= 1.500000
CONC. RATIO= .924918
STOKES NUMBER= .008000
CRITICAL ORD.= 1.177870 .000000

CORRECTED SIZE DISTRIBUTION

RADIUS(MICRONS)	FRACTION
3.5	0.093
7.5	0.261
12.5	0.223
26.5	0.340
51.0	0.070

GLOSSARY

A_i	Aspiration coefficient due to inertia
B	Non-dimensional function (Equation 8)
C	Measured concentration (#/cm^3)
C_o	Actual concentration (#/cm^3)
D	Length of line sink (cm)
d	Diameter of particle
E	Efficiency of capture
f	Fraction of diameter with line sink
F	Rate of suction of sample (cm^3/sec)
F_e	External force (gm·cm/sec^2)
Fr	Froude number ($V_S t/L$)
f_i	Fraction of i^{th} particle group
g	Acceleration due to gravity (cm/sec^2)
H	Half width of channel (cm)
I	Grid point along Z direction
J	Grid point along y or r direction
K	Stoke's number ($U_I t/L$)
L	Characteristic length (cm)
m	Sink/Source strength
n	Number of particle groups
P_e	Peclet number ($U_I L/E$)
Q_p	Electrostatic charge on particle (statcoulums)
Q_c	Electrostatic charge on collector (statcoulums)
R	Radius of the tube
R_o	Radial boundary of flow field
r	Cylindrical coordinate
S_c	Schmidt number
t	Time scale

u	Suction velocity of sample
U_I	Free stream velocity
$\overline{U}$	Flow velocity vector
U_x, U_y	Flow velocity along x + y direction
U_γ, U_z	Flow velocity along γ + z direction
$\overline{v}$	Particle velocity vector
V_s	Terminal settling velocity of particle
W	Width of sampler wall
x,y,z	Cartesian coordinates
Δy, Δz, $\Delta\gamma$	Grid size intervals in y, z, γ directions
ψ	Stream function
$\psi_{\mathbb{C}}$	Stream function at center line
α	Orientation of sampler face plane to the direction of flow
τ	Relaxation time of particles
η	Efficiency of sampling

*USGPO: 1981 — 757-074/1075

best copy available

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PUBLIC HEALTH SERVICE
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